ARYAN SCHOOL OF ENGINEERING & ECHNOLOGY

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LECTURE NOTE

SUBJECT NAME- STRUCTURAL DESIGN-I

BRANCH-CIVIL ENGG.

SEMESTER-4TH SEM

ACADEMIC SESSION-2022-23

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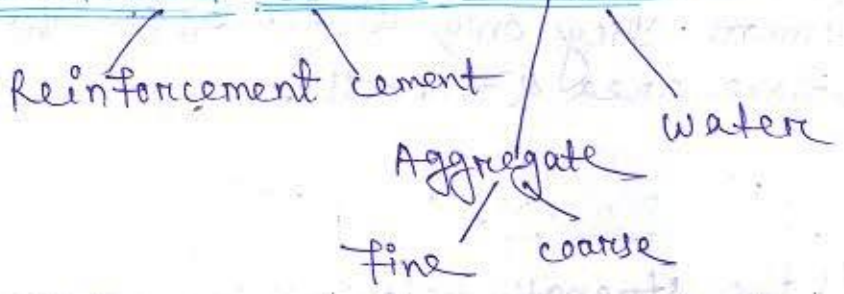
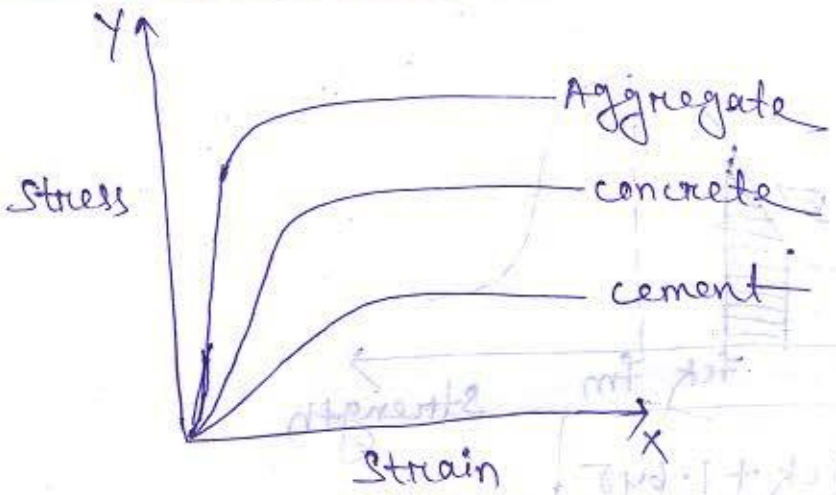
Reinforced cement concrete :-



It is the mixture of cement, fine aggregate, coarse aggregate & required quantity of water.

Stress-strain line :-



Cement :-

Cement was developed by Josfeh Aspdim.
Generally 3 grades of cement are available.

1. C_{33}
2. C_{43}
3. C_{53}

33 represents the compressive strength of cement mortar cube of size 7.07 mm at 28 days.

'C' represents the mixture of cement & sand.

Unit of 33, 43 & 53 is in MPa i.e. $33 N/mm^2$.

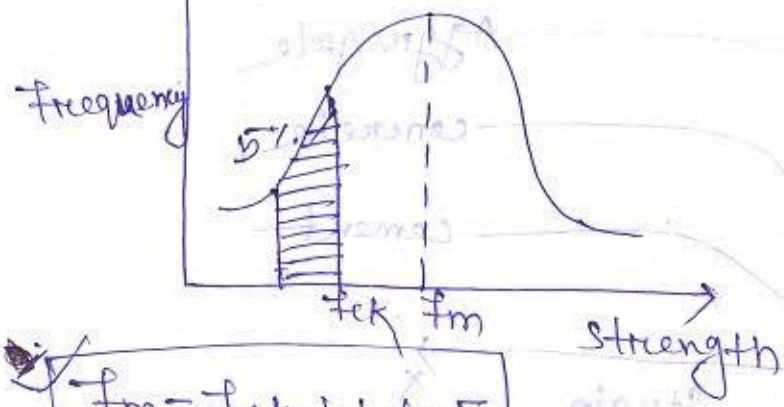
There is no difference between the different grades of cement. They only differ as per their specific surface area & fineness.

Characteristics strength (fck):—

It is the strength below which not more than 5% of test result are expected to fall.

If 100 cubes are tested atleast 95 cubes should pass the test result.

Relationship



$$f_m = f_{ck} + 1.64\sigma$$

σ = Standard deviation

f_m = Mean strength.

When no. of samples 'n' is > 30,

$$\sigma = \sqrt{\frac{\Sigma (f - f_m)^2}{n-1}}$$

When $n < 30$,

$$\sigma = \sqrt{\frac{\Sigma (f - f_m)^2}{n}}$$

Grade	σ
$M_{10} - M_{15}$	3.5
$M_{20} - M_{25}$	4
$M_{30} - M_{35}$	5

→ M_{25} represent the characteristics strength ($f_{ck} = 25\ N/mm^2$) of the mix 'M'.

Q:- Find out the target mean strength of M_{35}, M_{25} & M_{15} concrete.

Ans:- For M_{35} concrete,

$$f_m = f_{ck} + 1.64\sigma$$
$$= 35 + 1.64 \times 5$$
$$= 43.2\ N/mm^2$$

For M_{25} concrete,

$$f_m = f_{ck} + 1.64\sigma$$
$$= 25 + 1.64 \times 4$$
$$= 31.56\ N/mm^2$$

For M_{15} concrete,

$$f_m = f_{ck} + 1.64\sigma$$
$$= 15 + 1.64 \times 3.5$$
$$= 20.74\ N/mm^2.$$

Factors affecting the strength of concrete:-

Dimension of cylinder = 150 x 300 mm.

Dimension of the cube = 150 x 150 x 150 mm.

1. Shape:-

cylinder strength = 0.8 x cube strength
= 0.8 x f_{ck}.

cube strength = 1.25 x cylinder strength.

2. Size :—

The 10cm size cube strength is 5% more than that of 15cm cube due to better homoginity & quality control.

Water cement ratio :—

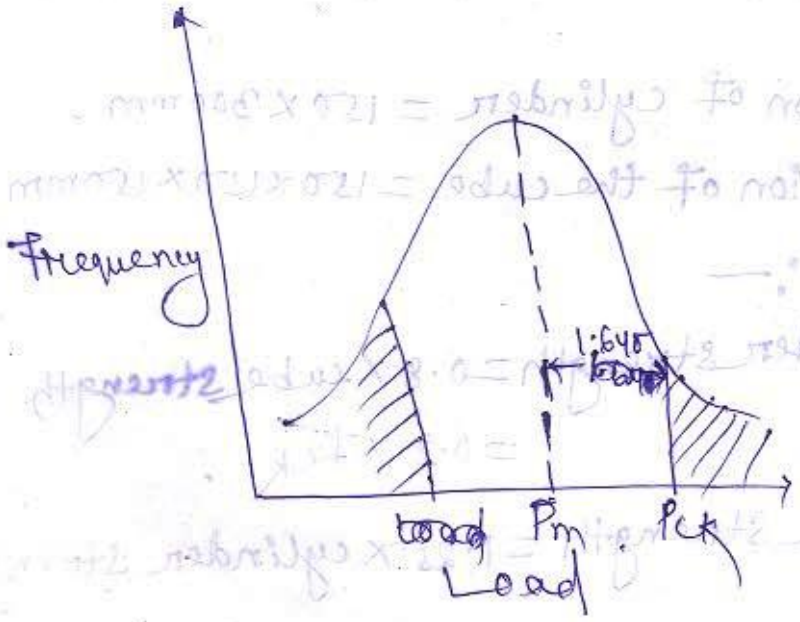
According to Abrahams law water cement ratio is inversely proportional to the strength of concrete that means if we decrease the water cement ratio then the strength of concrete will be enhanced.

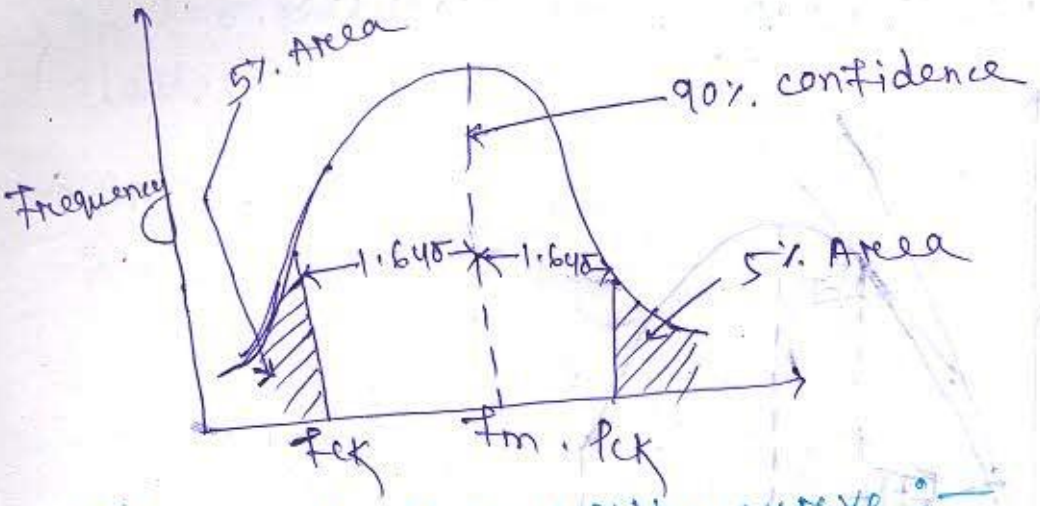
Characteristics load :—

It is the load which is having 95% Probability of not being exceeded during entire life of the structure.

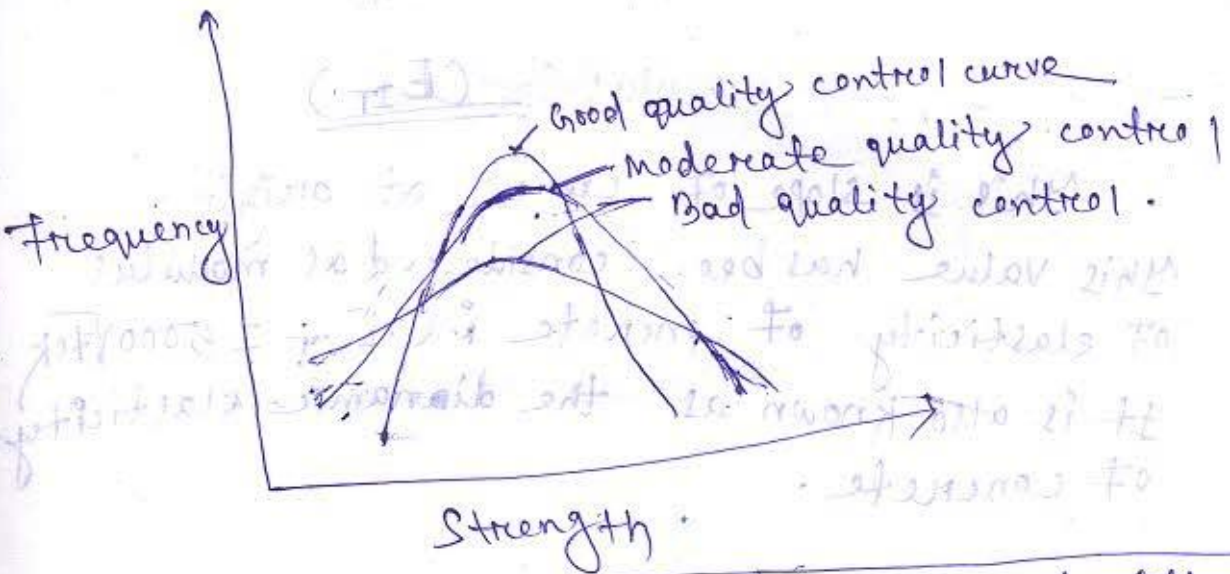
$$f_m = f_{ck} + 1.64\sigma$$

$$\boxed{P_m = P_{ck} + 1.64\sigma}$$





Different Probability curves:-



Design load = Load factor × characteristics strength.

Young's modulus of elasticity of concrete:-

→ It is represented as E_c.

2. $E_c = 5000\sqrt{f_{ck}}$

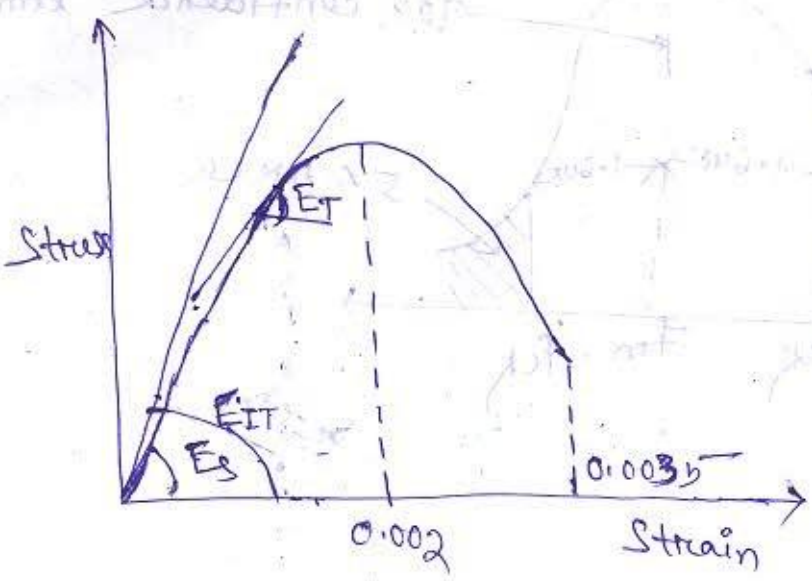
Q:- Find out the youngs modulus of M45 greade concrete ?

Ans:- $E_c = 5000\sqrt{f_{ck}}$

$= 5000\sqrt{45}$

$= 33541.02$ MPa.

Stress-strain diagram for concrete :—



→ **Initial Tangent modulus :— (E_{IT})**

This is slope of curve at origin. This value has been considered as modulus of elasticity of concrete i.e. $E_{IT} = 5000\sqrt{f_{ck}}$. It is also known as the dinamic elasticity of concrete.

→ **Second modulus of Elasticity (E_S) :—**

It is the slope of line joining any point on the curve with the origin.

→ **Tangent modulus of Elasticity (E_T) :—**

Slope of tangent at any point on the curve is called tangent modulus of elasticity.

→ In the linear region all the modulus of elasticity are same i.e. $E_{IT} = E_S = E_T$.

→ In non-linear region E_{IT} is greater than E_S is greater than E_T ($E_{IT} > E_S > E_T$).

Effect of creep on E_c :-

1. Time depended deformation (excluding strain due to shrinkage & temperature of total strain).
2. creep occures due to dead load only.

$$E_c = \frac{\sigma}{\varepsilon_c}$$

$E_{cc} = E_{ct}$ (long term effect)

$$E_{cc} = \frac{E_c}{1+\theta}$$

θ = creep co-efficient.

Tensile Strength of concrete :-

Direct tensile strength of the concrete cannot be measured because it is very difficult to perform the tensile strength test.

→ As the compressive strength increases the tensile strength also increases. But the rate of increase in compressive strength is more than that of the rate of increase in tensile strength.

2. Flexural Strength of the concrete :-

$$Fcr = 0.7 \times \sqrt{fck}$$

Split tensile strength :-

$$fct = \frac{2P}{\pi D L}$$

Where, P = Load

D = Diameter

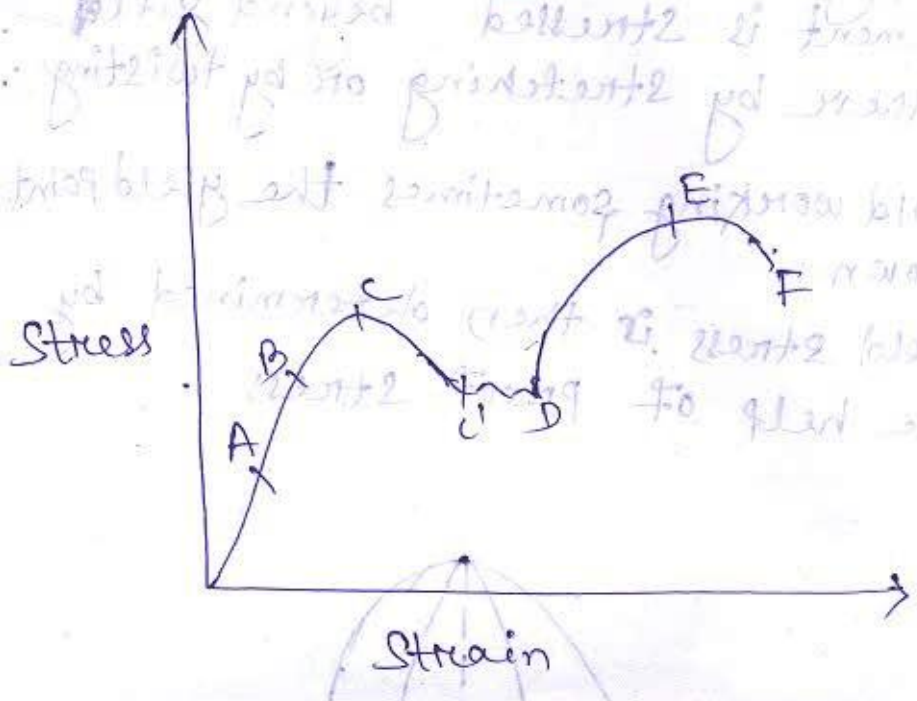
L = Length.

Steel or Reinforcement :-

The different types of reinforcement used in reinforced concrete are :

1. Mild steel
   (a) ordinary steel, (b) hot rolled steel.
2. Medium tensile steel.
3. High yield steel deformation bar (HYSD)
4. cold twisted deformed bar (CTD)
5. TMT bar (Thermo mechanically treated bar).

Stress-strain diagram of mild steel :—



A → Limit of Proportionality.
B → Elastic limit
C → Upper yield Point.
C' → Lower yield Point.
E → Ultimate Point.
F → Breaking Point.
DE → Strain hardening zone.
EF → Strain softening zone.
C'D → Yield Platue.

Hot rolled Mild Steel :—

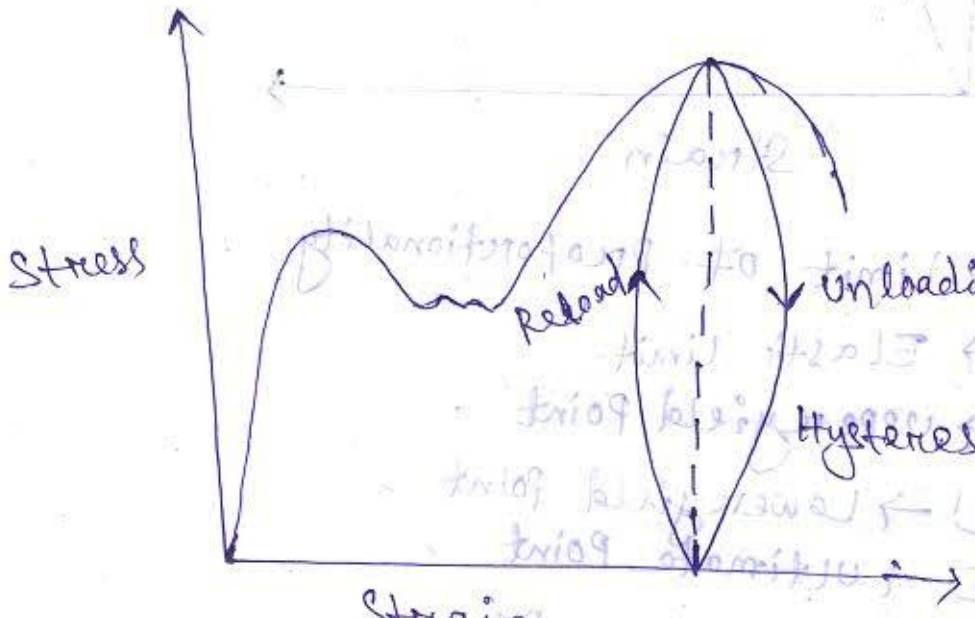
→ This reinforcement undergoes large deformation at yield platue resulting to large cracking on concrete structure.
This type of behaviour is not accepted.

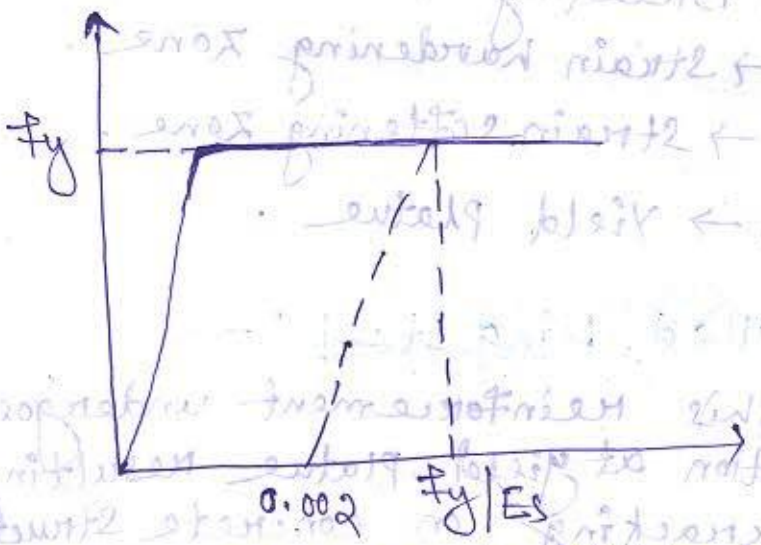
→ Yield platue can be avoided by cold working.

→ Cold working is the process in which reinforcement is stressed beyond yield Platue either by stretching or by twisting.

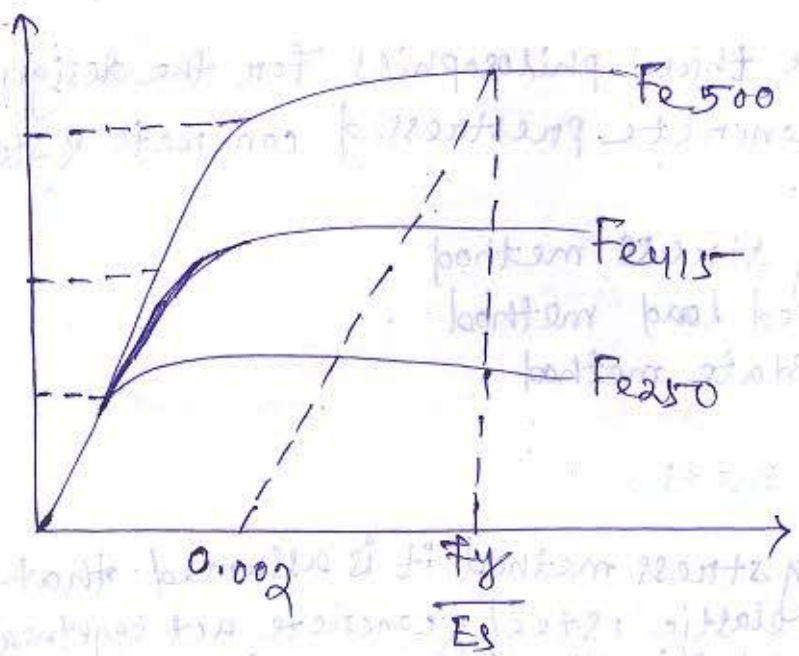
→ After cold working sometimes the yield point is not shown.

→ The yield stress is then determined by using the help of proof stress.



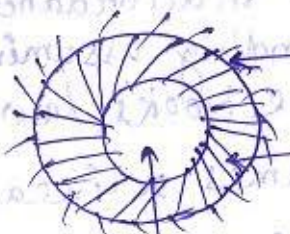


→ Modulus of elasticity of steel, $E_s = 2 \times 10^5$ MPa
$= 200$ GPa



TMT bar :-

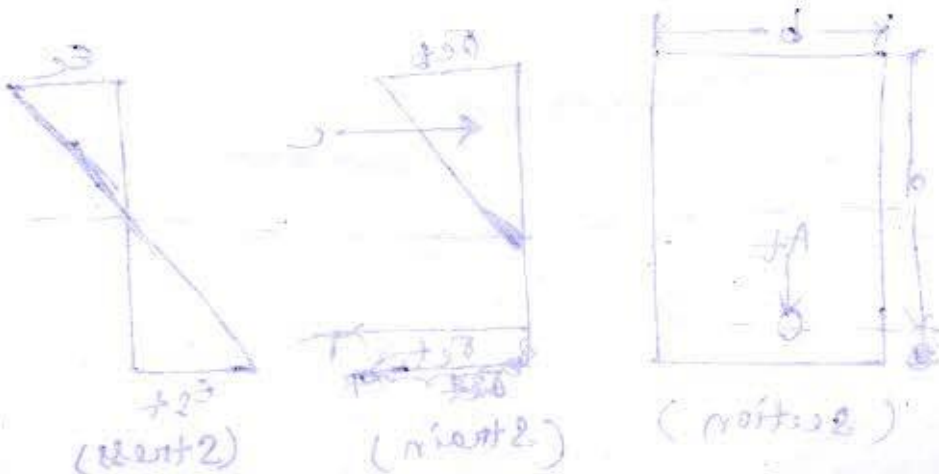
TMT bar is having both strength & ductility that is why it is mostly used now-a-days. When the cut end of TMT bars are dipped in nital solution (nitric acid + mithanol) three distinct layers appear.



Martensite, anticorrosive layer.

High strength, martensite + bainite

Ductile core, ferrite, martensite, anticorrosive layer bainite, perlite.



Design philosophies :—

There are three philosophies for the design of reinforced concrete prestressed concrete & steel structures.

1. Working stress method
2. Ultimated load method.
3. Limit state method.

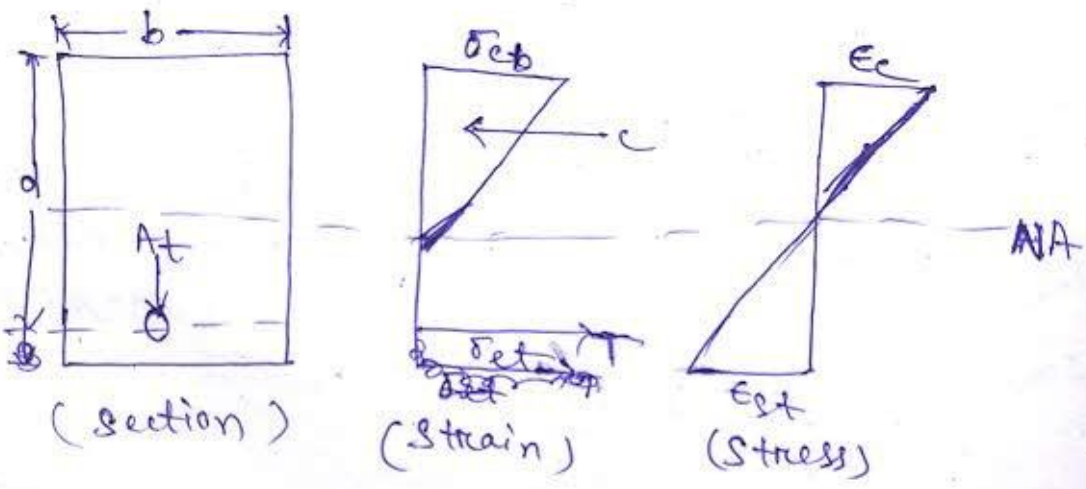
1. Working stress method :—

→ In working stress method it is assumed that concrete is elastic, steel & concrete act together elastically & the relationship between loads & stresses is linear right upto the collapse of the structure.

→ The basis of the method is that the permissible stress for concrete & steel are not exceeded any where in the structure when it is subjected to the worst combination of working loads.

→ The section is designed in accordance with the elastic theory of bending assuming that both materials obey the Hook's law.

→ The elastic theory assumes a linear variation of strain & stress from zero at neutral axis to a maximum at the extreme fibre.



A_1 = Area of tension steel

b = width of section

C = Total compressive force.

D = Depth of section

d = effective depth of section, defined as the depth from extreme compressive fibre to $c.c_1$ of tensile steel.

jd = Lever arm, defined as the distance between the point of application of force of compression & force of tension.

Nd = Depth of neutral axis.

T = Total force of tension.

σ_{cn} = permissible comp. stress in concrete.

σ_{st} = permissible tensile stress in steel.

E_c = compressive strain in concrete.

E_{st} = Tensile strain in steel.

Assumptions :—

(i) A section which is plane before bending remains plane after bending. This is also reffered to as Bernoulli's assumption.

(ii) Bond between steel & concrete is perfect within the elastic limit of steel.

(iii) Tensile strength of concrete is ignored.

(iv) Concrete is elastic, i.e. the stress in concrete varies linearly from zero at the neutral axis to a maximum at the extreme fibre.

(v) The modular ratio m has the value $\frac{280}{3\sigma_{cb}}$ where σ_{cb} is the permissible comp. stress in bending in N/mm^2 or MPa.

IS:456-2000 uses a factor of safety equal to 3, on the 28 days cube strength to obtain the permissible comp. stress in bending in concrete; & equal to 1.78 on the yield strength of steel in tension to obtain the permissible tensile strength in reinforcement. Thus for properly designed structural elements, the stresses computed under the action of working loads will be well within the elastic range.

Working stress method can be expressed as

$$MR \geq L$$

$$M = \frac{1}{\text{Factor of safety}}$$

which is always less than unity.

R - Resistance of the structural element.

L - working loads on the structural element.

Drawbacks of working stress method:-

(i) concrete is not elastic. The inelastic behaviour of concrete starts right from very low stresses.

(ii) The actual stress distribution in a concrete section can not be described by a triangular stress diagram.

(iii) Since factor of safety in on the stresses under working loads, there is no way to account for different degrees of uncertainty associated with different types of loads. with elastic theory it is impossible to determine the actual factor of safety with respect to loads.

(iv) It is difficult to account for shrinkage & creep effects by using the working stress method.

2. Ultimate load method :—

→ In ultimate load method the working loads are increased by suitable factors to obtain ultimate loads. These factors are called load factors. The structure is then designed to resist the desired ultimate loads. This method takes into account the non linear stress strain behaviour of concrete.

→ In working stress method factor of safety is defined as the ratio between yield stress & the working or permissible stress.

$$F.O.S = \frac{\text{Yield stress}}{\text{working stress}}$$

→ Load factor is defined as the ratio of collapse or ultimate load to the working load.

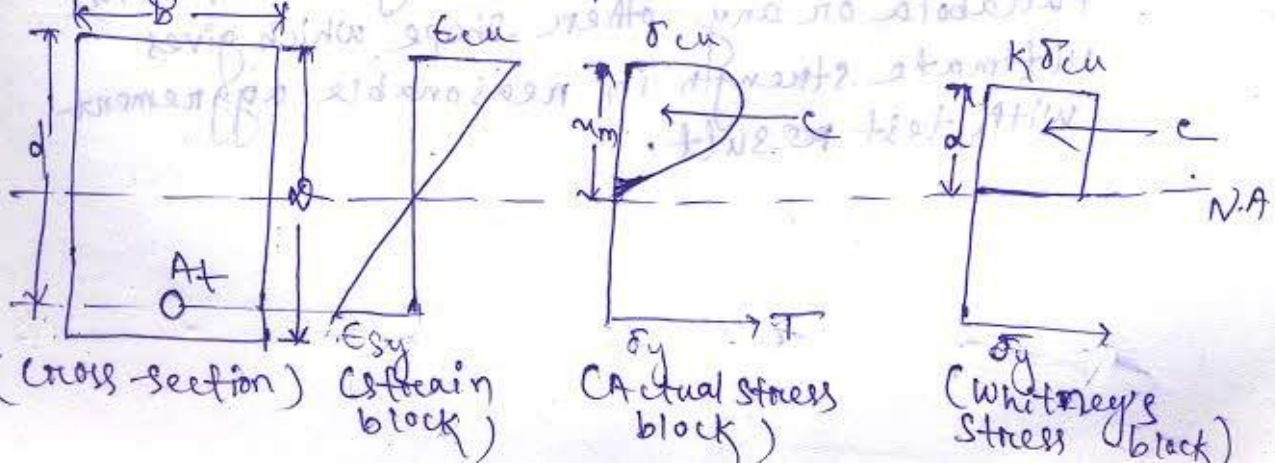
Assumptions of whitney's theory :—

(i) ultimate strain in concrete is 0.3%.

(ii) compressive stress at, extreme edge of the section corresponds to the ultimate strain.

(iii) Plane section before bending will remain plane after bending.

Whitney replaced the actual parabolic stress diagram by a rectangular stress diagram such that the c.g. of both the diagram lies at the same point & their areas are also equal. He found that the avg. stress of the rectangular stress diagram is equal to $k f_{cu}$.



Where,

a = depth of rectangular stress block.
= 0.537d in accordance with whitney
= 0.43d in accordance with IS: 456-1964

x_m = depth of Neutral axis at failure.

z = Lever arm.

δ_{cu} = Ultimate compressive strength of concrete cube at 28 days

$k\delta_{cu}$ = Avg. stress
= $0.85\delta_{cu}$ in accordance with whitney
= $0.55\delta_{cu}$ in accordance with is 456-1964.

δ'_{cu} = Ultimate compressive strength of concrete cylinder at 28 days.

δ_y = Yield stress in steel.

ϵ_{cu} = Ultimate strain in concrete

ϵ_{sy} = Yield strain in steel.

Assumption in accordance with IS: 456-1964:-

(i) A section which is plane before bending remains plane after bending.

(ii) An ultimate strength stress-strain are not Proportional & distribution of compressive stresses is non-linear in a section subjected to bending. The compressive stress diagram may by assumed as a rectangle, trapezoid, Parabola or any other shape which gives ultimate strength in reasonable aggrement with test result.

(iii) Maximum fibre strength in concrete does not exceeds $0.68 f_{cu}$. As in whitney's theory, the actual stress diagram can be replaced by a rectangular stress-block whose height 'a' is taken 0.43d & the avg. stress is assumed to be $0.53 f_{cu}$.

(iv) Tensile strength of concrete is ignored in sections subjected to bending.

The ultimate load - design method can be expressed as :

$$R > \lambda L$$

R - Resistance of the structural element.

L - Working loads on the structural element.

λ - Load factor which is more than unity.

A major advantage of this method over the working stress method is that total safety factor of a structure thus found in nearer to its actual value. Moreover the structures designed by the ultimate load method generally require less reinforcement than those designed by working stress method.

Drawbacks of Ultimate load Method :-

Main drawbacks of ultimate load method are as follows :

(i) Since load factor is used on the working loads. There is no way to account for different degree of uncertainties associated with variation in material stresses.

(ii) There is complete disregared for control against excessive deflection.

3. Limit state method :—

→ Limit state design has originated from ultimate or plastic design.

→ The object of design based on the limit state concept is to achieve an acceptable probability that a structure will not become unserviceable in its life time for the use for which it is intended, i.e. it will not reach a limit state.

→ A structure with appropriate degree of reliability should be able to withstand safety all loads that are liable to act on it throughout its life & it should also satisfy the serviceability requirement such as limitations on deflections & cracking.

→ It should be able to maintain the required structural integrity during & after accidents such as fires, explosions & local failure.

→ In other words all relevant limit states must be considered in design to ensure an adequate degree of safety & serviceability.

There are two methods of limit states.

1. Limit state of collapse
2. Limit state of serviceability.

Limit State of collapse :—

This state corresponds to the maximum load carrying capacity. violation of collapse limit states implies failure in the sense that a clearly defined limit state of structural usefulness has been exceeded. However it doesnot mean a complete collapse.

This limit state may correspond to

(a) Flexure.
(b) compression.
(c) Shear.
(d) Torsion.

Limit State of Serviceability :—

This state corresponds to development of excessive deformation & is used for checking members in which magnitude of deformations may limit the use of the structure or its components.

This limit state may correspond to

(a) Deflection
(b) cracking
(c) Vibration.

The choice of degree of reliability should take into account the possible consequences of exceeding the limit state of collapse which may be classified according to

(i) Risk to life is negligible & economic consequences small or negligible
(ii) Risk to life exists & / or economic consequences considered.
(iii) ~~Risk~~ Risk to life great & / or economic consequences also great.

→ Elastic theo theory or working stress theory is generally applicable for serviceability limit state & fatigue.

→ plastic theory for ultimate limit states & stability analysis for overturning.

→ In contrast or existing design methods limit states design applies to all kind of failure such as collapse overturning & vibration & to all materials & type of construction. In short limit state design of building Structures of all materials.

→ The limit state concept of design of reinforced concrete structures takes into account the Probabilistical & structural variation in material properties loads & safety factors.

Limit state of collapse can be expressed by the inequality.

$$MR > \sum_{i=1}^{n} \lambda_i L_i$$

→ Left hand side relates to the resistance or capacity of the Structure.
Right hand Side ~~relate~~ characterised the load acting on it.
The summation sign represents the combination of load effect from different load source, for example dead load, live load, wind or earthquake load.

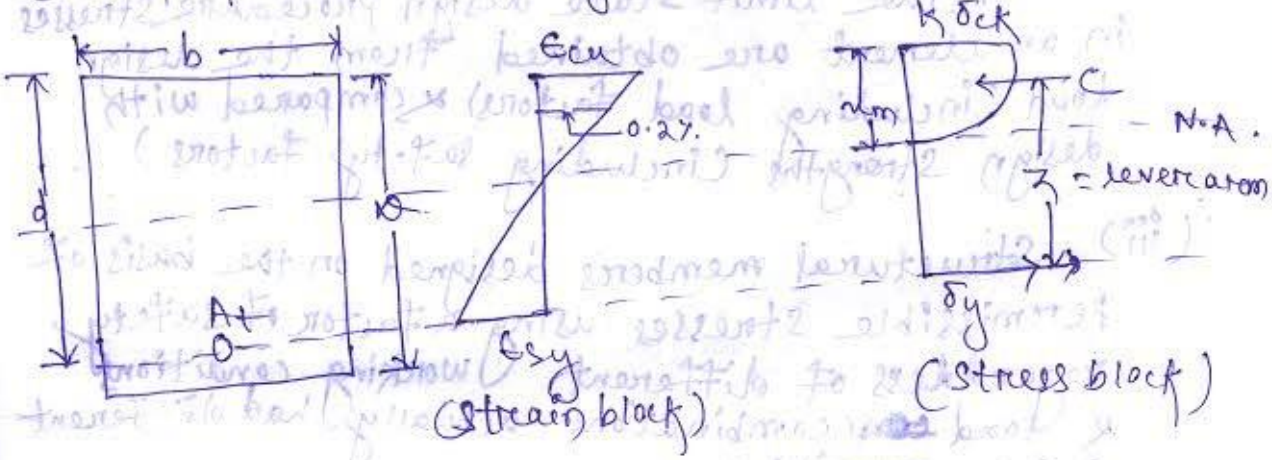
→ The randomness in the resistance R of a Structural element arises due to variation in material properties. workmanship & assumptions made in the theory underlying the design definition of member strength.

→ The safety factor μ which is always less than unity reflects the uncertainties associated with R.

→ The randomness in the evaluation of different loads L_i arises due to non-availability of sufficient & reliable data.

→ The load factor λ_i which is normally greater than unity.

In limit state concept of design of reinforced concrete structures, the factors μ & λ are called partial safety factors.



σ_{ck} = characteristics strength of concrete.

k = Safety factor.

The limit state of serviceability can be expressed by the inequality.

$$\frac{\delta}{L} \leq \frac{1}{\alpha}$$

δ = Deflection

L = Length or height or span of the structural element.

α = a non-dimensional number.

Limit State method Vs Working stress method :-

(i) Working stress method is reffered to as determinis because it is premused that loads, permissible stress & factor of safety are known accurately.

Limit state method is reffered to as Probabilistic because it is based on experience or on field data.

(ii) In working stress design method the stresses in an element are obtained from working load & compared with permissible stresses.

In the limit state design procedure stresses in an element are obtained from the design load (including load factors) & compared with design strengths (including safety factors).

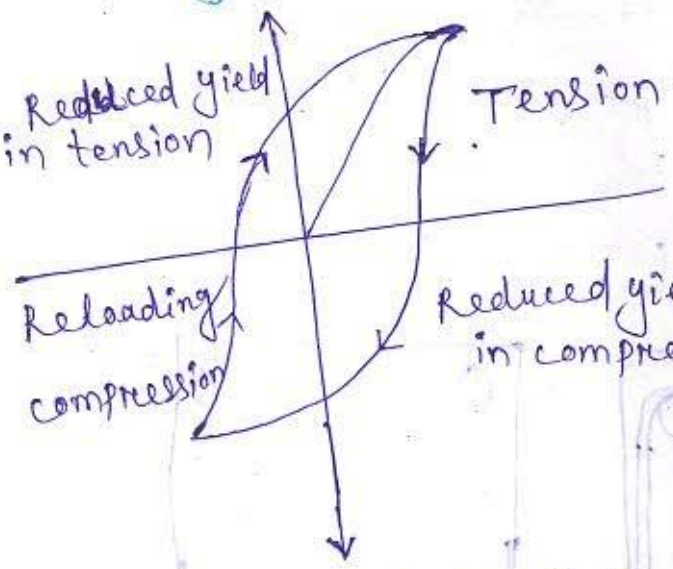
(iii) Structural members designed on the basis of permissible stresses using a factor of safety regardless of different working conditions & load combinations actually had different safety margins.

The limit state method is based on physical parameters. The partial safety factors are based on statistical it is a more scientific approach.

→ In limit state design method parameters are determined based on observations taken over period of time. These parameter will thus be influenced by chance or random effect not just at a single instant but throughout the entire period of time or the sequence of time that is being considered. Such process is known as stochastic process.

In a rough sense a stochastic process is a Phenomena that varies to some degree unpredict-ably as time goes on. It is also reffered to as a random process.

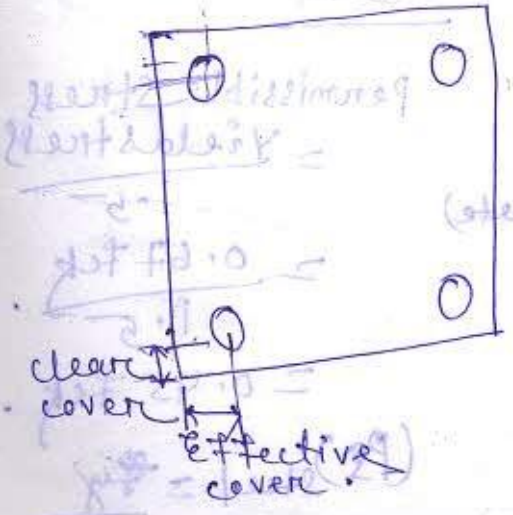
Bauchinger Effect :-



Stress-strain behaviour of mildsteel in compression is identical to that of compression.

However if the steel is stressed into the elastic range in uniform tension, unloaded & then subjected to uniform compression that is reverese loading. It is found that the stress-strain curve in compression becomes non-linear at a stress much lower than the initial yield strength.

Nominal cover 'or' clear cover :-



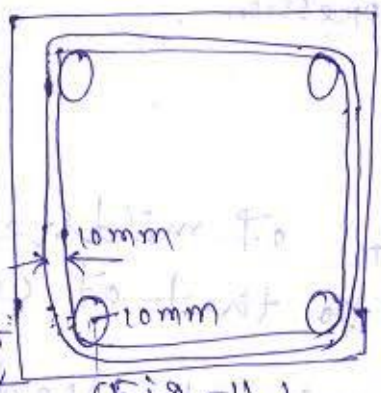
Nominal cover for mild exposure condition

Slab - 20mm .

Beam - 25 mm .

Column - 40 mm .

Footing - 50 mm .



(Fig-11)

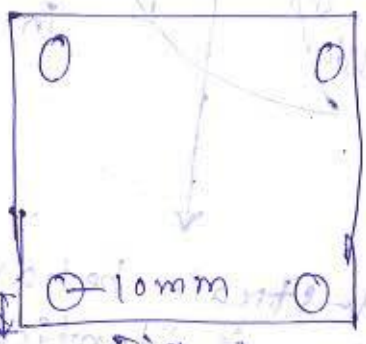


Fig-1

1. Effective cover $= 20 + \frac{10}{2} = 25$ mm.

2. Effective cover $= 20 + 10 + \frac{10}{2}$

$= 35$ mm.

Factor of Safety $= \frac{\text{Yield Stress}}{\text{Permissible stress}}$.

Permissible Stress $= \frac{\text{Yield Stress}}{\text{F.O.S}}$.

WSM

→ Permissible Stress $= \frac{\text{Yield stress}}{3}$ (concrete)

$= \frac{f_{ck}}{3}$.

→ Permissible stress in steel,

$(PS)_{steel} = \frac{f_y}{1.8}$

LSM

Permissible Stress $= \frac{\text{Yield stress}}{1.5}$

$= \frac{0.67 f_{ck}}{1.5}$.

$= 0.45 f_{ck}$.

$(PS)_{steel} = \frac{f_y}{1.15}$

$= 0.87 f_y$

Modular Ratio :—

It is the ratio of elasticity of steel to the elasticity of concrete.

It is represented by m.

$$m = \frac{E_s}{E_c} = \frac{2 \times 10^5}{5000\sqrt{f_{ck}}}$$ (without considering creep)

If we consider the creep,

$$m = \frac{2 \times 10^5}{5000\sqrt{f_{ck}}}(1+\theta)$$

θ = creep co-efficient.

As per IS:456

$$m = \frac{280}{3\sigma_{cbc}}$$

The partial safety factor for steel is less than that of concrete because steel is manufactured in factories under proper quality control, whereas concrete is manufactured in site. So the quality of concrete cannot be assured.

(Page-68-Table-18)

Design load = Load factor x characteristics load.

~~Dead load + Imposed load~~
~~(DL) (IL)~~

~~Dead load~~

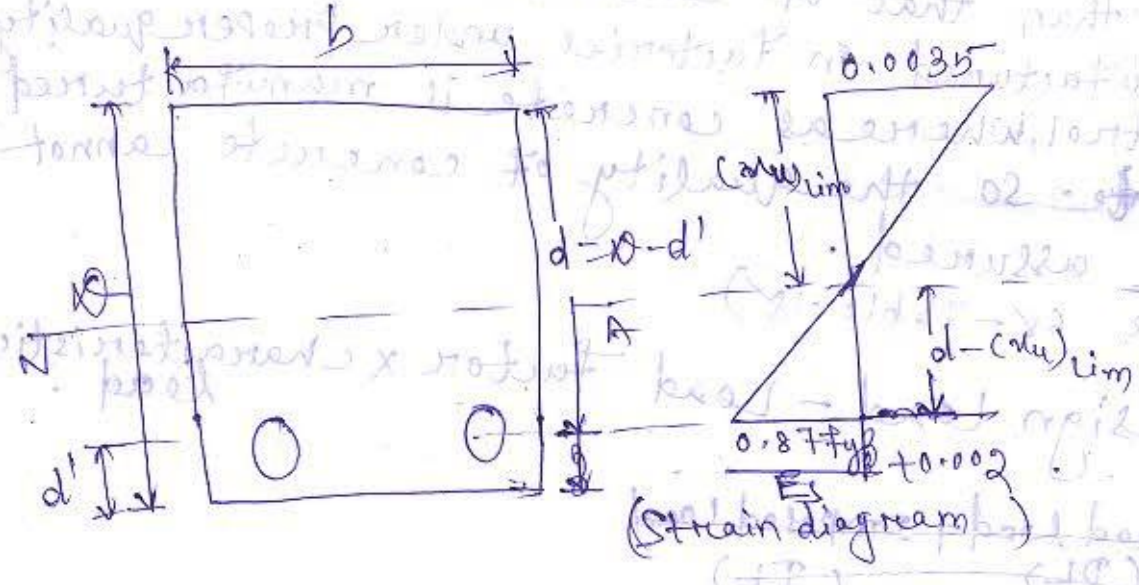
Limit state of collapse

	DL	LL	WL/EL
DL+LL/IL	1.5	1.5	
DL+WL/EL	1.5		1.5
DL+IL+WL	1.2	1.2	1.2

Limit state of serviceability

	DL	LL	WL/EL
DL+IL	1.0	1.0	—
DL+WL/EL	1.0	—	1.0
DL+IL+WL	1.0	0.8	0.8

Analysis of Singly reinforced beam:-



(Strain diagram)

$$\frac{d-(x_u)_{lim}}{(x_u)_{lim}} = \frac{\frac{0.87f_y}{E_s}+0.002}{0.0035}$$

$$\Rightarrow \frac{d}{(x_u)_{lim}} - 1 = \frac{\frac{0.87f_y}{E_s}}{0.0035} + \frac{0.002}{0.0035}$$

$$\Rightarrow \frac{d}{(x_u)_{lim}} - 1 = \frac{0.87 f_y}{0.0035 \times 2 \times 10^5} + \frac{0.002}{0.0035}$$

$$= 1.24 \times 10^{-3} f_y + 0.571 .$$

$$\Rightarrow \frac{d}{(x_u)_{lim}} = 1.24 \times 10^{-3} f_y + 0.571 + 1 .$$

$$\Rightarrow \frac{(x_u)_{lim}}{d} = \frac{1}{1.24 \times 10^{-3} f_y + 1.571}$$

$$\Rightarrow \frac{(x_u)_{lim}}{d} = \frac{700}{1100 + 0.87 f_y}$$

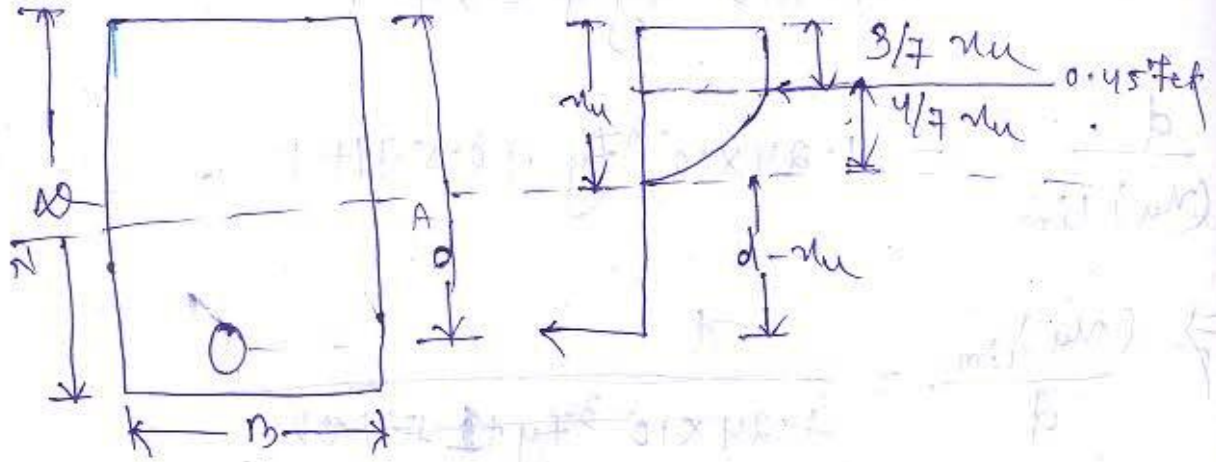
$$\Rightarrow \left(\frac{(x_u)_{lim}}{d}\right)_{Fe250} = \frac{700}{1100 + 0.87 \times 250}$$

$$= 0.53$$

Fe415 $\rightarrow \left(\frac{x_u}{d}\right)_{lim} = 0.48$.

Fe500 $\rightarrow \left(\frac{x_u}{d}\right)_{lim} = 0.46$.

Analysis of stress diagram :-



$C_1 = 0.45 f_{ck} \times A_1$

$= 0.45 f_{ck} \times B \times \frac{3}{7} x_u$.

$C_2 = 0.45 f_{ck} \times \frac{2}{3} \times \frac{4}{7} x_u \times B$.

$C = C_1 + C_2$

$= 0.45 f_{ck} \times \frac{3}{7} x_u \times B + 0.45 f_{ck} \times \frac{4}{7} x_u \times \frac{2}{3} x$

$= 0.45 f_{ck} \times x_u \times B \times \left(\frac{3}{7} + \frac{4}{7} \times \frac{2}{3}\right)$

$= 0.45 f_{ck} \times x_u \times B \times 0.809$

$= 0.36 f_{ck} x_u B$.

Force of steel $= 0.87 f_y \times A_{st}$

$0.36 f_{ck} x_u B = 0.87 f_y A_{st}$.

Moment $= 0.87 f_y A_{st} (d - 0.42 x_u)$ (Tensile)

$= 0.36 f_{ck} x_u B (d - 0.42 x_u)$ (compress

→ When $x_u < (x_u)_{lim}$, the beam is underreinforced.

→ when $x_u = (x_u)_{lim}$, the beam is a balanced section.

→ When $x_u > (x_u)_{lim}$, the beam is over-reinforced.

Restrict the value of x_u to $(x_u)_{lim}$.
or redesign the beam.

$$(M.O.R)_{lim} = 0.148 f_{ck} bd^2 \quad (Fe250)$$
$$= 0.138 f_{ck} bd^2 \quad (Fe415)$$
$$= 0.133 f_{ck} bd^2 \quad (Fe500)$$

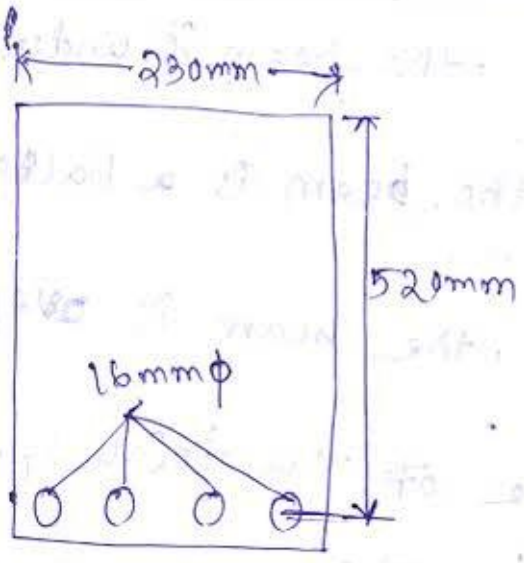
Formula

1. $p_t = \frac{A_{st}}{bd} \times 100$ (page-89)

2. $A_{st} = 0.5 \frac{f_{ck}}{f_y} \left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} bd^2}}\right] \times bd$.

Problem :-

A rectangular beam 230 mm wide 520 mm effective depth is reinforced with 4nos. of 16 mm dia bars. Find out the depth of nutral axis & specify the type of beam. The materials are M20 grade concrete & HYSD reinforcement of Fe415. Also find out the depth of nutral axis if the reinforcement is increased to 4no. of 20 mm dia bars.



$b = 230\ mm$

$d = 520\ mm$

$A_{st} = \frac{\pi}{4} \times d^2$

$= \frac{\pi}{4} \times (16)^2 \times 4$

$= 804.24 \simeq 804\ mm^2$

$f_{ck} = 20\ N/mm^2 = 20\ MPa$

$f_y = 415\ N/mm^2$

$C = T$

$\Rightarrow 0.36 f_{ck} x_u b = 0.87 f_y A_{st}$

$\Rightarrow x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$

$= \frac{0.87 \times 415 \times 804}{0.36 \times 20 \times 230}$

$= 175.3\ mm$

$\frac{(x_u)_{diam}}{d} = 0.48$

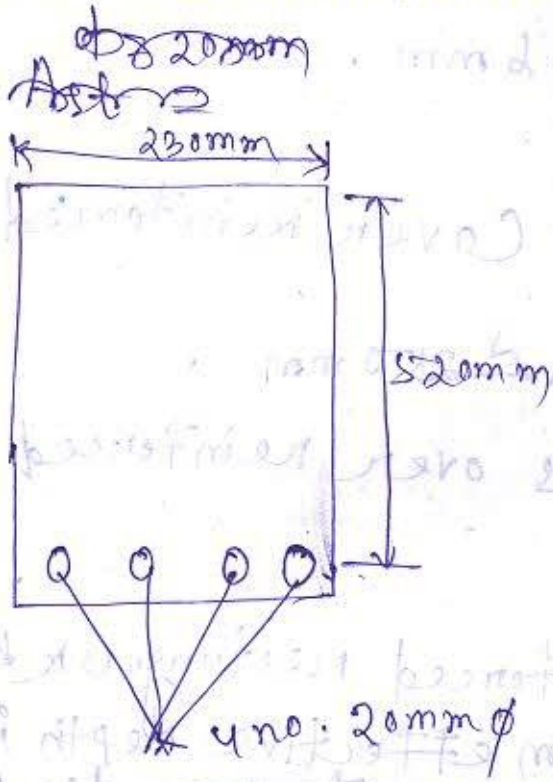
$\Rightarrow (x_u)_{diam} = 0.48 \times 520$

$= 249.6\ mm.$

$x_u < (x_u)_{lim}$ (under reinforced).

$x = 175$ mm.

So, the beam is under reinforced.



$b = 230$ mm

$d = 520$ mm

$A_{st} = \frac{\pi}{4} \times 20^2 \times 4$

$= 1256.64$

$\simeq 1257$ mm^2.

$f_{ck} = 20$ N/mm^2

$f_y = 415$ N/mm^2

$C = T$

$\Rightarrow 0.36 f_{ck} x_u b = 0.87 f_y A_{st}$

$\Rightarrow x_u = \frac{0.87 \times 415 \times 1257}{0.36 \times 20 \times 230}$

$= 274.1$ mm.

$$\frac{(x_u)_{lim}}{d} = 0.48$$

$$\Rightarrow (x_u)_{lim} = 0.48 \times 520$$

$$= 249.6 \text{ mm}.$$

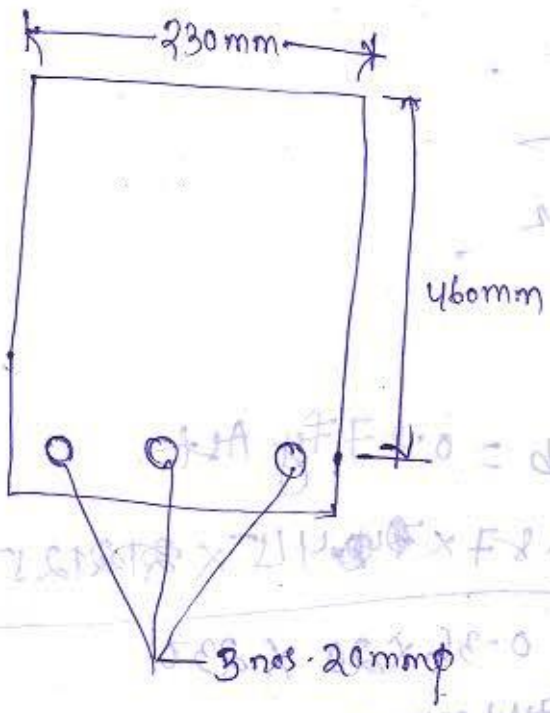
$x_u > (x_u)_{lim}$ (Over reinforced)

$x_u = 249.6 \text{ mm} \triangleq 250 \text{ mm}$.

So, the beam is over reinforced.

Q:- A singly reinforced rectangular beam of width 230 mm & 460 mm effective depth is reinforced with 3nos. of 20 mm dia bar. Find out the factored moment of resistance of the section, the materials are M20 grade concrete & Fe415 steel.

Sol:-



$b = 230\,mm$

$d = 460\,mm$

$A_{st} = 3 \times \frac{\pi}{4} \times 20^2$

$= 943\,mm^2$.

$f_{ck} = 20\,MPa$.

$f_y = 415\,MPa$.

$C = T$

$\Rightarrow 0.36 f_{ck} x_u b = 0.87 f_y A_{st}$

$\Rightarrow x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$

$= \frac{0.87 \times 415 \times 943}{0.36 \times 20 \times 230}$

$= 206\,mm$.

$\frac{(x_u)_{lim}}{d} = 0.48$

$\Rightarrow (x_u)_{lim} = 0.48 \times d$

$= 0.48 \times 460$

$= 221\,mm$.

$x_u < (x_u)_{lim}$.

So the the section is under reinforced .

$x_u = 206\,mm$.

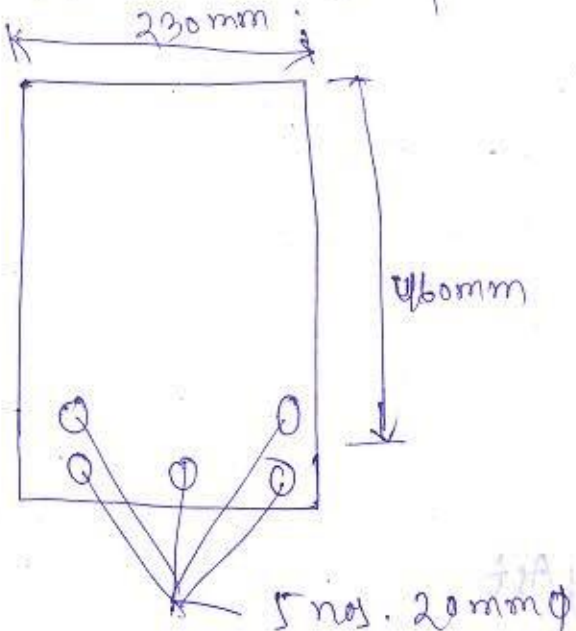
$M.O.R = 0.87 f_y A_{st} (d - 0.42 x_u)$

$= 0.87 \times 415 \times 943 (460 - 0.42 \times 206)$

$= 127158741.6\,Nmm$.

$= 127.15\,KNm$.

No. of bar = 5



$b = 230\ mm$

$d = 460\ mm$

$f_{ck} = 20\ MPa$

$f_y = 415\ MPa$

$$A_{st} = \frac{\pi}{4} \times 20^2 \times 5$$

$$= 1570.79\ mm^2$$

$$= 1571\ mm^2$$

Compression = Tension

$$\Rightarrow 0.36 f_{ck} x_u b = 0.87 f_y A_{st}$$

$$\Rightarrow x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 1571}{0.36 \times 20 \times 230}$$

$$= 342.52\ mm$$

$$= 343\ mm.$$

$\frac{(x_u)_{lim}}{d} = 0.48$

$\Rightarrow (x_u)_{lim} = 0.48 \times d.$

$= 0.48 \times 460$

$= 220.8 \text{ mm}.$

$\simeq 221 \text{ mm}.$

M.O.R. $x > (x_u)_{lim}$.

So the section is over reinforced,

$x_u = 221 \text{ mm}.$

M.O.R for compression $= 0.36 f_{ck} x_u b (d - 0.42 x_u)$

$= 0.36 \times 20 \times 221 \times 230 (460 - 0.42 \times 221)$

$= 134379067.7 \text{ Nmm}$

$= 134.37 \text{ KNm}.$

M.O.R for tension $= 0.87 f_y A_{st} (d - 0.42 x_u)$

$= 0.87 \times 415 \times 1571 \times (460 - 0.42 \times 221)$

$= 208268002.6 \text{ Nmm}$

$= 208.26 \text{ KNm}.$

Q:- A singly reinforced beam is subjected to a bending moment of 36 KNm at working load. The width of the beam is 200 mm. Find the depth & steel area for balanced design. The material is M_{20} concrete & mildsteel.

$f_{ck} = 20 Mpa$.

$f_y = 250 Mpa$.

$b = 200 mm$.

working moment $= 36 kNm$.

Factored moment or Design moment $= 36 \times 1.5$

$= 54 kNm$.

$= 54 \times 10^6 N\text{-}mm$.

$(M_u)_{lim} = 0.148 f_{ck} b d^2$.

$\Rightarrow 54 \times 10^6 = 0.148 \times 20 \times 200 \times d^2$

$\Rightarrow d = \sqrt{\frac{54 \times 10^6}{0.148 \times 20 \times 200}}$

$= 302.02$

$\simeq 302 mm$.

$$A_{st} = 0.5 \frac{f_{ck}}{f_y} \left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}\right] \times bd$$

$$= 0.5 \times \frac{20}{250} \left[1 - \sqrt{1 - \frac{4.6 \times 54 \times 10^6}{20 \times 200 \times (302)^2}}\right] \times 200 \times$$

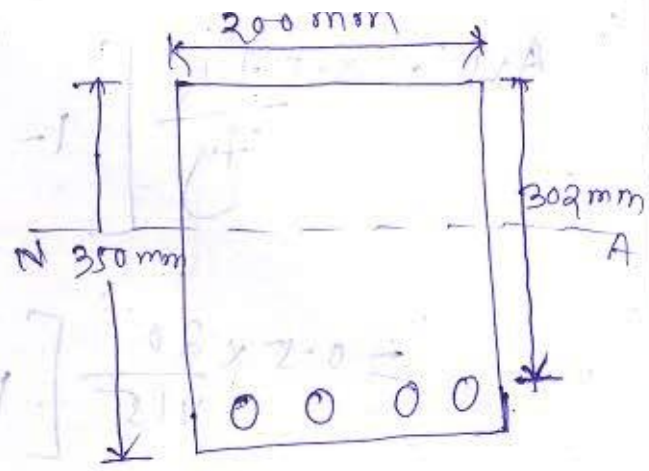
$= 1051 mm^2$

$n \times \frac{\pi}{4} \times 20^2 = 1051 mm^2$

$\Rightarrow n = \frac{1051 \times 4}{\pi \times 20^2}$

$= 3.34 \simeq 4 nos$.

4 nos. of 20 mm ϕ bar



Overall depth,

$D = 302 + \frac{20}{2} + 25$

$= 337$ mm

≈ 350 mm.

Q:- Design a singly reinforced rectangular beam for an applied factored moment of 120 KNm. Assume the width of the section 230 mm. The materials are M20 grade concrete & Fe415 Steel.

Soln

Factored moment, $M_u = 120$ KNm $= 120 \times 10^6$ N-mm.

width, $b = 230$ mm

$f_{ck} = 20$ N/mm^2

$f_y = 415$ N/mm^2.

$(M_u)_{lim} = 0.138 f_{ck} b d^2$

$\Rightarrow 120 \times 10^6 = 0.138 \times 20 \times 230 \times d^2$

$\Rightarrow d = \sqrt{\frac{120 \times 10^6}{0.138 \times 20 \times 230}}$

$= 434.78$ mm

≈ 440 mm.

$$A_{st} = 0.5 \frac{f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}\right] x b d$$

$$= 0.5 \times \frac{20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 120 \times 10^6}{20 \times 230 \times 440^2}}\right] \times 230 \times 440$$

$$= 935.00 \text{ mm}^2$$

Let us provide 16mm dia bars

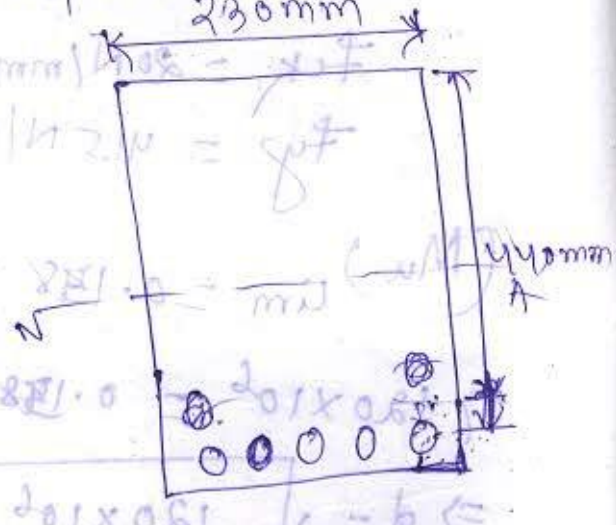
$$n \times \frac{\pi}{4} \times 16^2 = 935$$

$$\Rightarrow n = 935 \times \frac{4}{\pi} \times \frac{1}{16^2}$$

$$= 4.65$$

$$\simeq 5 \text{ nos.}$$

Provide 5 nos. of 16mm ϕ bar.



Overall depth,

$$D = 440 + \frac{16}{2} + 25$$

$$= 473$$

$$\simeq 480 \text{ mm.}$$

Let us provide 20 mm dia bars,

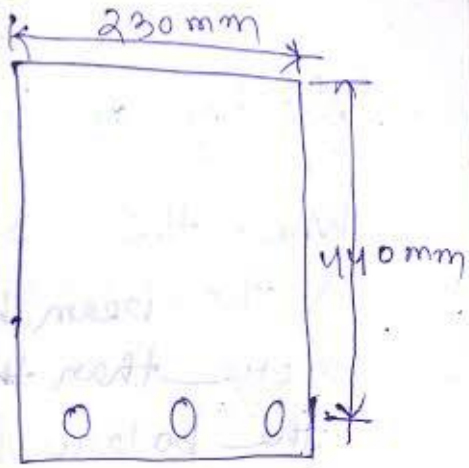
$$n \times \frac{\pi}{4} \times 20^2 = 935$$

$$\Rightarrow n = 935 \times \frac{4}{\pi} \times \frac{1}{20^2}$$

$$= 2.97$$

$$\simeq 3 \text{ nos.}$$

Provide 3 nos. of 20mm ϕ bar



Over all depth

$= 440 + \frac{20}{2} + 25$

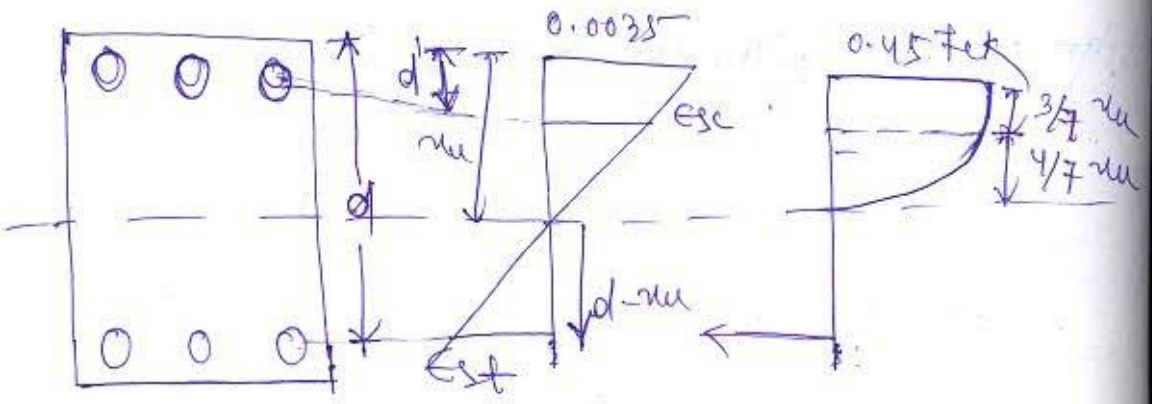
$= 475$

$\simeq 480$ mm

~~Design of doubly reinforced beam~~

Design of Doubly reinforced beam:-

When the size of the beam is restricted restrict & the beam has to ~~resist more~~ moment more than the moment of resistance of the balanced section then we need a doubly reinforced beam.



$$\frac{\epsilon_{st}}{0.0035} = \frac{d-x_u}{x_u}$$

$$\Rightarrow \epsilon_{st} = 0.0035\left(\frac{d}{x_u} - 1\right)$$

$$\frac{\epsilon_{sc}}{0.0035} = \frac{x_u - d'}{x_u}$$

$$\Rightarrow \epsilon_{sc} = 0.0035\left(1 - \frac{d'}{x_u}\right)$$

$$C = T$$

$$0.36 f_{ck} x_{ub} - 0.45 f_{ck} A_{sc} + A_{sc} f_{sc} = f_{st} A_{st}$$

$$f_{sc} = f_{st} = 0.87 f_y.$$

$0.36 f_{ck} x_{u_b} + A_{sc}(f_{sc} - 0.45 f_{ck}) = f_{st} A_{st}$.

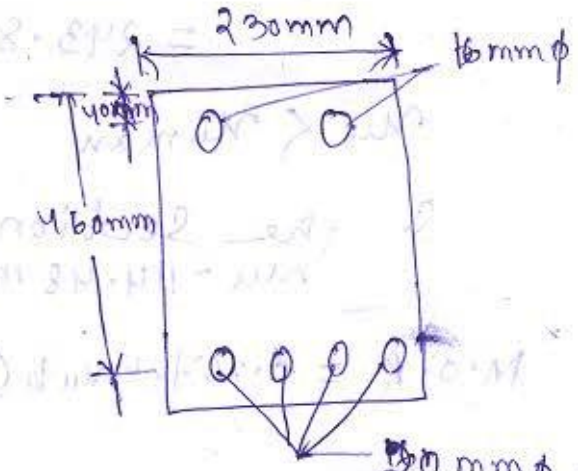
M.O.R For compression :-

$M.O.R = 0.36 f_{ck} x_{u_b} (d - 0.42 x_u) + A_{sc}(f_{sc} - 0.45 f_{ck})(d - d')$

HW Page - 6-5, 7, 8, 9

Q:- Find the factored moment of resistance of a beam section 230mm wide & 460mm effective depth with two 16mm dia bar as compression reinforcement & the at an effective cover of 40mm & four 20mm diameter bar as tension reinforcement. Use M20 grade concrete & mild steel reinforcement.

Sol:-



$b = 230\,mm$

$d = 460\,mm$.

$d' = 40\,mm$.

$A_{sc} = 2 \times \frac{\pi}{4} \times 16^2$

$= 402.12\,mm^2 \simeq 402\,mm^2$

$A_{st} = 4 \times \frac{\pi}{4} \times 20^2$

$= 1256.63\,mm^2$,

$\simeq 1257\,mm^2$

$f_{ck} = 20\,MPa$.

$f_y = 250\,MPa$.

$C = T$

$0.36 f_{ck} x_{u,lim} b + A_{sc}(f_{sc} - 0.45 f_{ck}) = 0.87 f_y A_{st}$

$\Rightarrow x_u = \dfrac{0.87 f_y A_{st} - A_{sc}(f_{sc} - 0.45 f_{ck})}{0.36 f_{ck} b}$

$= \dfrac{0.87 \times 250 \times 1257 - 402(0.87 \times 250 - 0.45 \times 20)}{0.36 \times 20 \times 230}$

$= 114.48 \text{ mm}$.

$\dfrac{x_{u,max}}{d} = 0.53$ (Page – 70)

$\Rightarrow x_{u,max} = 0.53 \times d$

$= 0.53 \times 460$

$= 243.8 \text{ mm}$.

$x_u < x_{u,max}$.

So, the section is under reinforced.

$x_u = 114.48 \text{ mm}$.

$M.O.R = 0.36 f_{ck} x_{u,lim} b (d - 0.42 x_u) + A_{sc}(f_{sc} - 0.45$

$= 0.36 \times 20 \times 114.48 \times 230 (460 - 0.42 \times 114.48)$

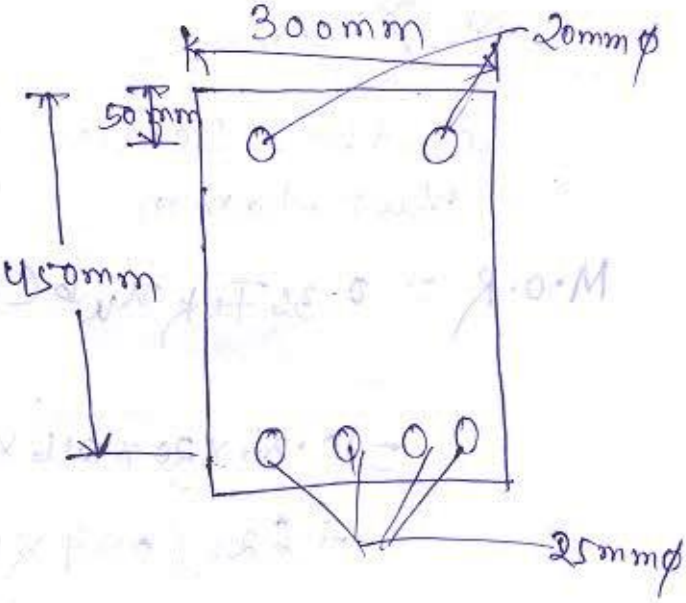
$+ 402(0.87 \times 250 - 0.45 \times 20)(460 - 40$

$= 113294168.9 \text{ Nmm}$

$= 113.29 \text{ kNm}$.

Q:- Find out the factored moment of resistance of a beam having 300mm wide & 450mm effective depth reinforced with two 20 mm diameter bar as compression reinforcement ~~a tension reinforcement the materials are~~ at an effective cover of 50 mm & four 25 mm dia bar as tension reinforcement, the material are M20 & HYSD bar.

Soln



$b = 300\text{mm}$

$d = 450\text{mm}$

$d' = 50\text{mm}$

$f_{ck} = 20\text{ MPa}$

$f_y = 415\text{ MPa}$

$A_{sc} = 2 \times \frac{\pi}{4} \times 20^2$

$= 628.31\text{ mm}^2$

$\simeq 628\text{ mm}^2$

$A_{st} = 4 \times \frac{\pi}{4} \times 25^2$

$= 1963.49\text{ mm}^2$

$= 1964\text{ mm}^2$

$C = T$

$0.36 f_{ck} x_u b + A_{sc}(f_{sc} - 0.45 f_{ck}) = 0.87 f_y A_{st}$

$\Rightarrow x_u = \frac{0.87 f_y A_{st} - A_{sc}(f_{sc} - 0.45 f_{ck})}{0.36 f_{ck} b}$

$= \frac{0.87 \times 415 \times 1964 - 628(0.87 \times 415 - 0.45 \times 20)}{0.36 \times 20 \times 300}$

$= 225.93$

$\simeq 226\text{ mm}$

$\frac{x_{umax}}{d} = 0.48$

$\Rightarrow x_{umax} = 0.48 \times d$

$= 0.48 \times 450$

$= 216 \text{ mm}$.

$x_u > x_{umax}$

So, the section is over reinforced.

$x_u = 216 \text{ mm}$

$M.O.R = 0.36 f_{ck} x_u b (d - 0.42 x_u) + A_{sc} (f_{sc} - 0.45 f_{ck}) (d - d')$

$= 0.36 \times 20 \times 216 \times 300 (450 - 0.42 \times 216)$

$+ 628 (0.87 \times 415 - 0.45 \times 20) (450 - 50)$

$= 256060636.8 \text{ Nmm}$

$= 256.06 \text{ kNm}$.

Date - 08/02/2019

~~$(M_u)_{lim}$~~.

$M_u = (M_u)_{lim} + M_{u_2}$ (Page - 96)

$\Rightarrow M_{u_2} = M_u - (M_u)_{lim}$

$M_{u_2} = f_{sc} A_{sc} (d - d')$

Q:- Design the doubly reinforced rectangular beam of size 230mm wide 500mm effective depth is subjected to a factored moment 200 kNm. Find the reinforcement for flexure. The materials are M20 grade concrete & HYSD bar of Fe415.

Sol:-

Given data,

$b = 230mm$

$d = 500mm$

$M_u = 200 kNm$

$f_{ck} = 20 MPa$

$f_y = 415 MPa$

Let us assume $d' = 50mm$

$$(M_u)_{lim} = 0.138 f_{ck} b d^2$$

$$= 0.138 \times 20 \times 230 \times 500^2$$

$$= 158.7 kNm$$

$$M_{u2} = M_u - (M_u)_{lim}$$

$$= 200 - 158.7$$

$$= 41.3 kNm = f_{sc} A_{sc} (d - d')$$

$$41.3 = 0.87 f_y A_{sc} (d - d')$$

$$\Rightarrow 41.3 = 0.87 \times 415 \times A_{sc} (500 - 50)$$

$$\Rightarrow A_{sc} = \frac{41.3 \times 10^6}{0.87 \times 415 (500 - 50)}$$

$$= 254.196 mm^2$$

$$\simeq 260 mm^2$$

Let us provide 16 mm dia.

4:36

$$n \times \frac{\pi}{4} \times 16^2 = 260$$

$$\Rightarrow n = 260 \times \frac{4}{\pi} \times \frac{1}{16^2}$$

$$= 1.29$$

$$\simeq 2 \text{ nos}.$$

Let us provide 2 – 16 mm dia bar.

$$A_{sc} = 2 \times \frac{\pi}{4} \times 16^2 \quad 2 \times \frac{\pi}{4} \times 16^2$$

$$= 804.24 \quad = 402.12 \text{ mm}^2$$

$$\simeq 804 \text{ mm}^2. \quad \simeq 402 \text{ mm}^2$$

$$(A_{st})_2 = \frac{f_{sc} A_{sc}}{0.87 f_y}$$

$$= \frac{0.87 f_y A_{sc}}{0.87 f_y}$$

$$= A_{sc} = 260 \text{ mm}^2.$$

$$(M_u)_{lim} = 0.87 f_y (A_{st})_1 \left(d - 0.42 (x_u)_{lim}\right)$$

$$\frac{(x_u)_{lim}}{d} = 0.48$$

$$\Rightarrow (x_u)_{lim} = 0.48 \times 500$$

$$= 240 \text{ mm}.$$

$$\Rightarrow 158.7 \times 10^6 = 0.87 \times 415 (A_{st})_1 (500 - 0.42 \times$$

$$\Rightarrow (A_{st})_1 = \frac{158.7 \times 10^6}{0.87 \times 415 \times (500 - 0.42 \times 240)}$$

$$= 1101.08$$

$$\simeq 1110 \text{ mm}^2.$$

$A_{st} = (A_{st})_1 + (A_{st})_2$

$= 1110 + 402$

$= 1512\ mm^2$.

Let us use 20 mm dia bar,

$n \times \frac{\pi}{4} \times 20^2 = 1512$

$\Rightarrow n = 1512 \times \frac{4}{\pi} \times \frac{1}{20^2}$

$= 4.81$

$\simeq 5$ no.

$A_{st} = 5 \times \frac{\pi}{4} \times 20^2$

$= 1570.79\ mm^2$

$\simeq 1570\ mm^2$.

$C = T$

$0.36 f_{ck} x_u b + A_{sc}(f_{sc} - 0.45 f_{ck}) = 0.87 f_y A_{st}$

$\Rightarrow x_u = \frac{0.87 f_y A_{st} - A_{sc}(f_{sc} - 0.45 f_{ck})}{0.36 \times f_{ck} b}$

$= \frac{0.87 \times 415 \times 1570 - 402(0.87 \times 415 - 0.45 \times 20)}{0.36 \times 20 \times 230}$

$= 256.84$

$x_u > x_{u,lim}$

The section is over reinforced,

Let us assume 4 nos. of 16 mm ϕ bar at compression side.

$$A_{sc} = 4 \times \frac{\pi}{4} \times 16^2$$

$$= 804.24 mm^2$$

$$\simeq 804 mm^2$$

$$0.36 f_{ck} x_u b + A_{sc}(f_{sc} - 0.45 f_{ck}) = 0.87 f_y A_{st}$$

$$\Rightarrow x_u = \frac{0.87 f_y A_{st} - A_{sc}(f_{sc} - 0.45 f_{ck})}{0.36 \times f_{ck} \times b}$$

$$= \frac{0.87 \times 415 \times 1570 - 804(0.87 \times 415 - 0.45 \times 2}{0.36 \times 20 \times 230}$$

$$= 171.37 mm.$$

$x < (x_u)_{lim}$,

So, the section is under reinforced.

Flanged beam :—

(Page – 37 ~~3~~ 93.1.2)

(a) For T-beams, $b_f = \frac{l_o}{6} + b_w + 6D_f$ ⎫
(b) For L-beams, $b_f = \frac{l_o}{12} + b_w + 3D_f$ ⎭ For monolythically casted beam.

For isolated beams,

$$T\text{-beam}, b_f = \frac{l_n}{\left(\frac{l_o}{b}\right)+4} + b_w$$

$$L\text{-beam}, b_f = \frac{0.5 l_n}{\left(\frac{l_o}{b}\right)+4} + b_w$$

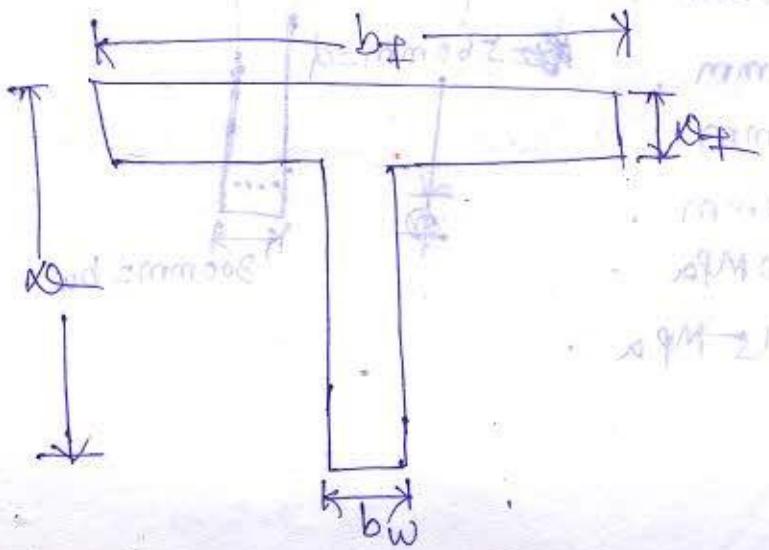
Where, b_f = effective width of flange.

l_o = Distance between points of zero moments.

b_w = breadth of the web.

D_f = thickness of the flange.

b = Actual width of the flange.



$C = 0.36 f_{ck} b_f D_f$.

$T = 0.87 f_y A_{st}$.

When $C < T$, the nutreal axis is ~~above~~ in the web.

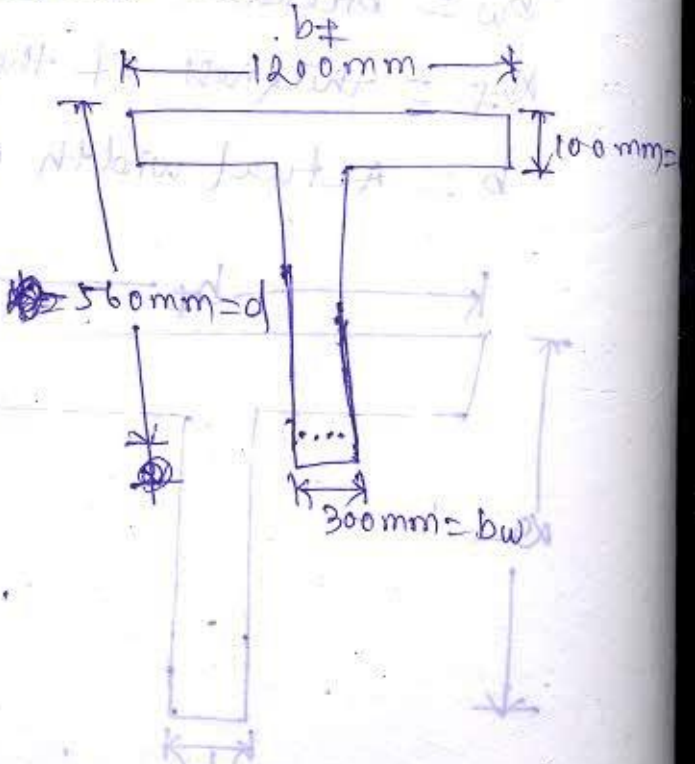
When $C = T$, the nutreal axis will lie at the bottom of the flange.

When $C > T$, the nutreal axis lie in the flange section.

Date - 12/02/2019

Q:- A T-beam of effective flange width of 1200mm, thickness of the slab 100mm, width of the rib 300 mm effective depth of 560mm is reinforced with 4no. of 25 mm dia HYSD Fe415 bar. calculate the factored moment of resistance, the grade of concrete is M_{20}.

Soln



$b_f = 1200mm$.

$D_f = 100mm$.

$d = 560mm$

$b_w = 300mm$.

$f_{ck} = 20 MPa$.

$f_y = 415 MPa$.

$A_{st} = 4 \times \frac{\pi}{4} \times 25^2$

$= 1963.49$

$\simeq 1964 \text{ mm}^2$.

$F_{tc} = 0.36 f_{ck} b_f d_f$

$= 0.36 \times 20 \times 1200 \times 100$

$= 864000 \text{ N}$

$= 864 \text{ KN}$.

$F_{ts} = 0.87 f_y A_{st}$

$= 0.87 \times 415 \times 1964$

$= 709102.2 \text{ N}$

$= 709 \text{ KN}$

$F_{tc} > F_{ts}$ (N.A lies in the flange).

$0.36 f_{ck} x_u b_f = 0.87 f_y A_{st}$

$\Rightarrow x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$

$= \frac{0.87 \times 415 \times 1964}{0.36 \times 20 \times 1200}$

$= 82.07$

$\simeq 82 \text{ mm}$.

$\left(\frac{x_u}{d}\right)_{lim} = 0.48$

$\Rightarrow x_{ulim} = 0.48 \times d$

$= 0.48 \times 560$

$= 268.8$

$\simeq 269 \text{ mm}$.

$x_u < (x_u)_{lim}$

So, the beam is under reinforced.

$$\frac{D_f}{d} = \frac{100}{560}$$

$$= 0.17 < 0.2$$

$$M_u = 0.36 \frac{x_{umax}}{d}\left[1 - 0.42 \frac{x_{umax}}{d}\right] f_{ck} b_w$$

$$+ 0.45 f_{ck} (b_f - b_w) D_f \left(d - \frac{D_f}{2}\right)$$

$$= 0.36 \times \frac{269}{560}\left[1 - 0.42 \times \frac{269}{560}\right] \times 20 \times 300 \times$$

$$+ 0.45 \times 20 \times (1200 - 300) \times 100 \left(560 - \frac{100}{2}\right)$$

$$= 672836500.8 \text{ Nmm}$$

$$= 672.83 \text{ kNm}$$

→ Instead of 560mm to take 450 mm.

$A_{st} = 1964 \text{ mm}^2$

$d = 450 \text{ mm}$

$x_u = 82.07$

$$\frac{(x_u)_{lim}}{d} = 0.48$$

$$\Rightarrow (x_u)_{lim} = 0.48 \times 450$$

$$= 216$$

$$\frac{x_{umax}}{d} = \frac{100}{450}$$

$$= 0.22 > 0.2$$

~~$M_{u,lim} = 0.36$~~

$$y_f = 0.15 x_u + 0.65 x_{umax}$$

$$= 0.15 \times 82.07 + 0.65 \times 100$$

$$= 77.31 \text{ mm}$$

$$M_u = 0.36 \times \frac{x_{umax}}{d} \left(1 - 0.42 \frac{x_{umax}}{d}\right) f_{ck} b_w d^2 +$$

$$+ 0.45 f_{ck} (b_f - b_w) y_f \left(d - \frac{y_f}{2}\right)$$

$$= 0.36 \times \frac{216}{450} \times \left(1 - 0.42 \times \frac{216}{450}\right) \times 20 \times 300 \times 450^2 +$$

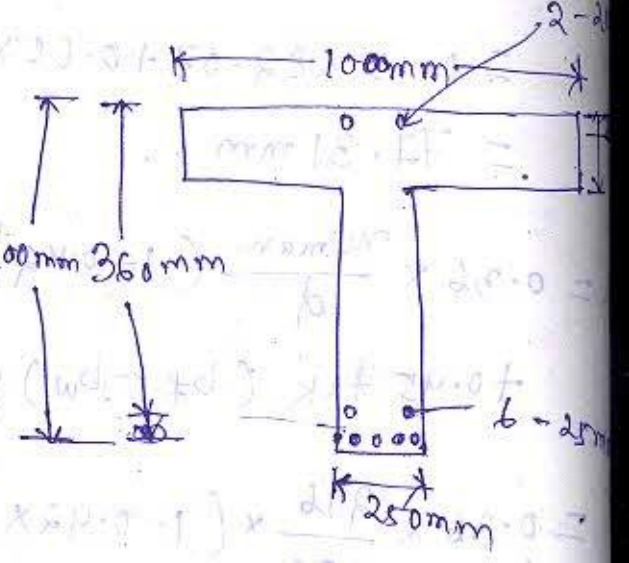
$$+ 0.45 \times 20 \times (1200 - 300) \times 77.31 \times \left(450 - \frac{77.31}{2}\right)$$

$$= 167625676.8 + 257588763.8$$

$$= 425214440.6 \text{ Nmm}$$

$$= 425.21 \text{ kNm}$$

Q:- Determine the moment of resistance of the section as shown in figure. The material are M20 grade concrete & HYSD bar of Fe415.



Sol:-

$f_{ck} = 20 MPa$
$f_y = 415 MPa$
$b_f = 1000mm$
$D_f = 100mm$
$d' = 40mm$
$d = 360mm$
$D = 400mm$
$b_w = 250mm$

$A_{sc} = 2 \times \frac{\pi}{4} \times 20^2$

$= 628.32 mm^2$

$\approx 628 mm^2$

$A_{st} = 6 \times \frac{\pi}{4} \times 25^2$

$= 2945.24 mm^2$

$\approx 2945 mm^2$.

$C = T$

$0.36 f_{ck} x_u b_f + A_{sc}(f_{sc} - 0.45 f_{ck}) = f_{st} A_{st}$

$\Rightarrow 0.36 \times 20 \times x_u \times 1000 + 628(0.87 \times 415 - 0.45 \times 20) = 0.87 \times 415 \times 2945$

$\Rightarrow x_u = \frac{0.87 \times 415 \times 2945 - 628(0.87 \times 415 - 0.45 \times 20)}{0.36 \times 20 \times 1000}$

$\Rightarrow x_u = 116.97$

$\simeq 117 \text{ mm}$.

$\left(\frac{x_u}{d}\right)_{lim} = 0.48$

$\Rightarrow (x_u)_{lim} = 0.48 \times d$

$= 0.48 \times 360$

$= 172.8 \text{ mm}$.

$\simeq 173 \text{ mm}$.

$x_u < (x_u)_{lim}$ (under reinforced)

$\frac{D_f}{d} = \frac{100}{360} = 0.28 > 0.2$.

$y_f = (0.15 x_u + 0.65 D_f)$

$= 0.15 \times 117 + 0.65 \times 100$

$= 82.55 \text{ mm}$.

$$M_u = 0.36 \frac{x_{umax}}{d}\left(1 - 0.42 \frac{(x_u)_{max}}{d}\right) f_{ck} b_w d^2 + 0.45 f_{ck} (b_f - b_w) y_f \left(d - \frac{y_f}{2}\right) + A_{sc} f_{sc} (d - d')$$

$$= 0.36 \times \frac{173}{360}\left(1 - 0.42 \times \frac{173}{360}\right) \times 20 \times 250 \times 360^2 + 0.45 \times 20 (1000 - 250) \times 82.55 \left(360 - \frac{82.55}{2}\right) + 628 \times 0.87 \times 415 (360 - 40)$$

$= 89477676 + 177597554.1 + 72557608$

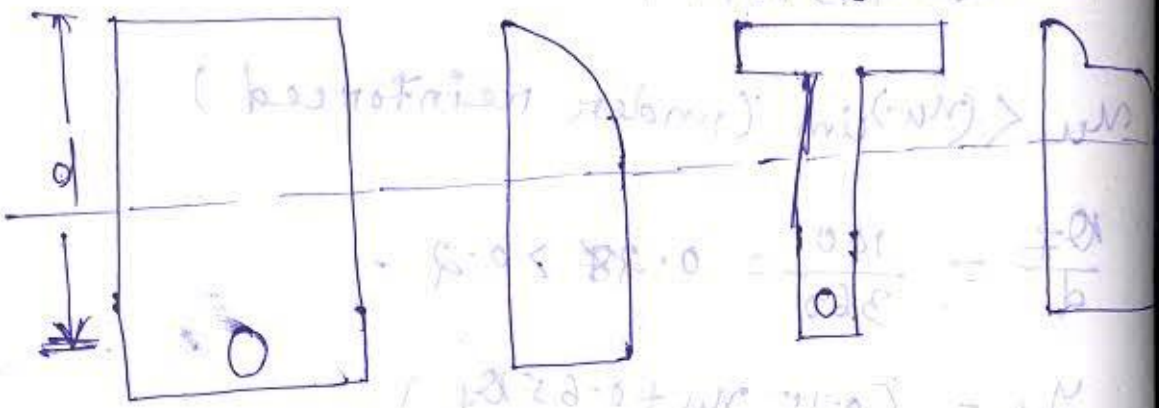
$= 339631838.1 \text{ Nmm}$

$= 339.63 \text{ kNm}$.

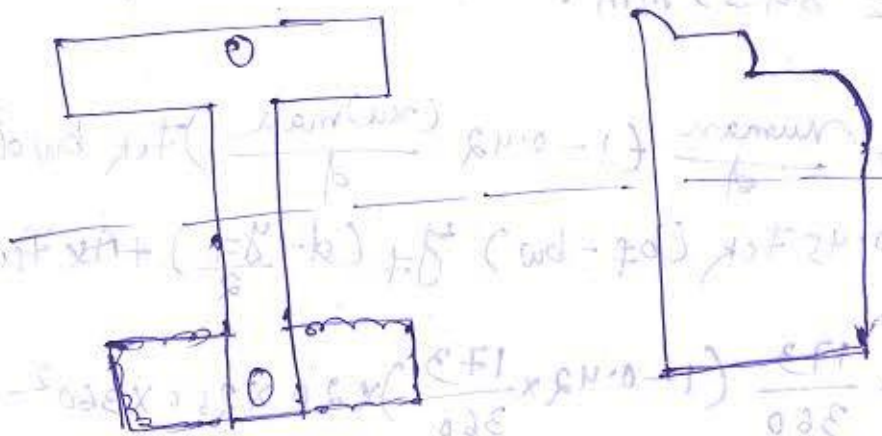
Design for shear

Shear stress distribution for a singly reinforced rectangular beam varies parabolically upto the nutral axis & then after remains constant upto the effective depth d.

For a singly reinforced flanged section



For a ~~singly~~ doubly reinforced flanged section



At the bottom fibre of any beam throughout the length shear stress is zero & bending stress is maximum. The nature of bending stress is tensile due to which ~~st~~ crack is formed at the bottom which is having an angle

of 90° with the beam axis.

Design Steps for Shear :—

Step-1

Nominal shear stress $= \tau_v = \frac{V_u}{bd}$

For flange section,

$$\tau_v = \frac{V_u}{b_w d}$$

Step-2

(Page-733, Table-20)

Check $\tau_v < \tau_{cmax}$.

If $\tau_v > \tau_{cmax}$ redesign the beam or improve the grade of concrete.

Shear strength of concrete depends on the following factors :

1. Uncracked concrete in compression zone.
2. Aggregate interlocking.
3. Shear acting along the longitudinal bar.
4. Shear forces across the steel bar.
5. Shear stirrups.

Step-3

Find out percentage of steel.

$$P_t = \frac{A_{st}}{bd} \times 100 \quad \text{(Page-73)}$$

From % of steel, calculate the value of τ_c from table-19.

Step-4

→ Check if $\tau_v < \frac{\tau_c}{2}$; no shear reinforcement is required.

→ If $\tau_v > \frac{\tau_c}{2}$ & $\tau_v < \tau_c$, the shear reinforcement is provided as per clause — 26.5.1.6.

$$\frac{A_{sv}}{b s_v} \geq \frac{0.4}{0.87 f_y}$$

~~A_{sv} = total cross-se No. of leg into $\frac{\pi}{4} \times \phi^2$~~

$$A_{sv} = \text{No. of leg} \times \frac{\pi}{4} \times \phi^2$$

Higher grade of steel Fe500 is restricted as a shear reinforcement because the area of steel required for higher grade steel is less due to which the ductility reduces.

Q:- Why minimum shear reinforcement is required?

Ans:-

1. To resist any crack development due to creep & shrinkage.
2. To improve the ductility of the beam.
3. To improve dowel action of main reinforcement.
4. To resist diagonal tension.

Step-5

If $\tau_v > \tau_c$, then

$$V_{us} = V_u - \tau_c bd$$

$$= \tau_v bd - \tau_c bd$$

$$= bd(\tau_v - \tau_c)$$

$$\tau_v = \frac{V_u}{bd}$$

7. Inclined stirrups are more effective in resisting shear as it is provided at an angle 90° to the propagation of crack.

Step-6

(Page-47 - 26.5.1.5-)

$S_v =$

$0.75d$ - vertical

d - inclined

300mm which ever is the less for spacing.

Date - 16/02/2019

Q:- A T-beam section having 230 mm width of the web & 460mm effective depth is reinforced with 5 no. of 16 mm dia bar as tension reinforcement. The section is subjected to a factored shear force of 52.5 KN. Check the shear stress & design the shear reinforcement, the materials are M_{20} & HYS 10 bar of Fe415 for stirrup use mildsteel.

→ What will be changed in shear reinforcement if the factored shear is increased to 90 KN & 6mm dia stirrups are used.

Soln

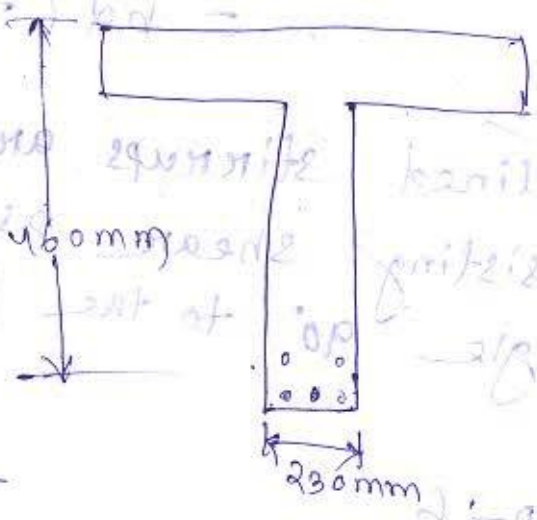
Given data

$b_w = 230\ mm$

$d = 460mm$

$f_{ck} = 20\ MPa$

$f_y = 415\ MPa$



$A_{st} = 5 \times \frac{\pi}{4} \times 16^2$

$= 1005.31\ mm^2$

$\simeq 1006\ mm^2$.

$V_u = 52.5\ KN = 52.5 \times 10^3\ N$.

$$\tau_v = \frac{V_u}{bd} = \frac{52.5 \times 10^3}{230 \times 460} = 0.49\ N/mm^2.$$

$\tau_v < (\tau_{c\,max}$ (safe).

$$p_t = \frac{100 A_{st}}{bd}$$

$$= \frac{100 \times 1006}{230 \times 460}$$

$$= 0.95$$

So calculate τ_c,

$$\tau_c = \frac{x - x_1}{x_2 - x_1} \times y_2 + \frac{x - x_2}{x_1 - x_2} \times y_1$$

x_1 0.75 — 0.56 y_1
x 0.95 — y
x_2 1.0 — 0.62 y_2

$$= \frac{0.95 - 0.75}{1 - 0.75} \times 0.62 + \frac{0.95 - 1}{0.75 - 1} \times 0.56$$

$$= 0.608$$

$$\frac{\tau_c}{2} = \frac{0.608}{2} = 0.304$$

$\tau_v < \frac{\tau_c}{2}$, $\tau_v < \tau_c$.

$\frac{\tau_c}{2} < \tau_v < \tau_c$.

$0.304 < 0.49 < 0.608$.

Minimum shear reinforcement will be provided,

$$\frac{A_{sv}}{b s_v} \geq \frac{0.4}{0.87 f_y}$$

For stirrups, $f_y = 250 \text{ N/mm}^2$.

A_{sv} = No. of leg $\times \frac{\pi}{4} \times \phi^2$.

Let us provide 2 leg, 6mm dia stirrup.

$$A_{sv} = 2 \times \frac{\pi}{4} \times 6^2$$

$$= 56.54 \text{ mm}^2$$

$$\approx 57 \text{ mm}^2$$.

$$\frac{A_{sv}}{bS_v} \geq \frac{0.4}{0.87 f_y}$$

$$\Rightarrow \frac{57}{230 \times S_v} \geq \frac{0.4}{0.87 \times 250}$$

$$\Rightarrow S_v \leq \frac{0.87 \times 250 \times 57}{0.4 \times 230}$$

$$\Rightarrow S_v = 134.75 \text{ mm}$$

≈ 130 mm

(a) $S_v = 130$ mm

(b) $0.75d = 0.75 \times 460 = 345$ mm

(c) 300 mm

Let us provide 2 leg 6mm dia stirrups with 130 mm c/c spacing.

Case-2

$V_u = 90 \text{ kN} = 90 \times 10^3 \text{ N}$

$$\tau_v = \frac{V_u}{bd} = \frac{90 \times 10^3}{230 \times 460}$$

$$= 0.85 \text{ N/mm}^2$$

$\tau_c = 2.8 \text{ N/mm}^2$

$\tau_v < (\tau_c)_{max}$ (Safe)

$$p_t = \frac{100 A_{st}}{bd} = \frac{100 \times 1006}{230 \times 460} = 0.95$$

$$\tau_c = \frac{0.95 - 0.75}{1 - 0.75} \times 0.62 + \frac{0.95 - 1}{0.75 - 1} \times 0.56$$

$$= 0.608 \text{ N/mm}^2$$

Now, we find

$\tau_v > \tau_c$.

Shear reinforced will be provided.

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

Here $f_y = 250$

$V_{us} = V_u - \tau_c bd$

$= 90 \times 10^3 - 0.608 \times 230 \times 460$

$= 25673.6$ N.

$= 25.6$ KN

Let us provide 2 leg 6 mm dia stirrups

$A_{sv} = 2 \times \frac{\pi}{4} \times 6^2$

$= 56.55 mm^2$.

For vertical stirrups,

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

$$\Rightarrow 25673.6 = \frac{0.87 \times 250 \times 56.55 \times 460}{S_v}$$

$$\Rightarrow S_v = \frac{0.87 \times 250 \times 56.5 \times 460}{25673.6}$$

$= 220.18$ mm.

$\simeq 220$ mm

The spacing shall not exceeds

(a) $0.75 \times d = 0.75 \times 460 = 345$ mm.

(b) 300 mm.

(c) 220 mm.

Let us provide 2 leg - 6 mm dia stirrups with

Date - 26/02/2018

Design for Bond :-

When reinforcing bar is embedded in concrete the concrete adheres to its surface & resist any force that tries to cause slippage of bar related to its surrounding concrete. This phenomena is called Bond.

Factors affecting development of bond stress

1. Pure adhesion.
2. Friction resistance
3. Mechanical resistance.

Bond stress in plain bar is due to pure adhesion & frictional resistance while in deformed bar bond stress is due to pure adhesion, frictional resistance & mechanical resistance. This is why bond stress of deformed bar is more in compare to plain bar.

(Page - 49, 26.2)

The development bond length, Page -

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$$

Where, ϕ = nominal diameter of the bar

σ_s = stress in bar at the section considered design load

$= 0.87 f_y$

τ_{bd} = design bond stress.

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}$$

(Page – 43, 26.2.1.1)

Q:- Findout the permissible stress of HYSD bar in tension & compression for M_{20} grade concrete.

Soln)

For M_{20} grade concrete, $\tau_{bd} = 1.2 \text{ N/mm}^2$.

In tension, $\tau_{bd} = 1.2 \times 1.6$
$= 1.92 \text{ N/mm}^2$.

In compression, $\tau_{bd} = 1.2 \times 1.6 \times 1.25$
$= 2.4 \text{ N/mm}^2$.

Q:- Findout the development length required for Fe415 & M_{25} grade concrete.

(i) In tension
(ii) In compression.
(iii) Also findout L_d in above case if mild steel is used.

Soln)

For M_{25} grade concrete,

$\tau_{bd} = 1.4 \text{ N/mm}^2$.

(i) ~~In compression.~~

(i) in tension $\tau_{bd} = 1.4 \times 1.6 = 2.24 \text{ N/mm}^2$.

$\sigma_s = 0.87 f_y$
$= 0.87 \times 415$

Development length, $L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$

$= \frac{\phi \times 361.05}{4 \times 2.24}$

$\Rightarrow L_d = 40.29 \phi$.

(ii) In compression, $\tau_{bd} = 1.4 \times 1.6 \times 1.25$

$= 2.8 \text{ N/mm}^2$.

Development length, $L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$

$= \frac{\phi \times 361.05}{4 \times 2.8}$

$\Rightarrow L_d = 32.23 \phi$.

(iii) For mild steel, $f_y = 250 \text{ N/mm}^2$.

$\tau_{bd} = 1.4$.

In tension

Development length, $L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$

$\Rightarrow L_d = \frac{\phi 0.87 f_y}{4 \times 1.4}$

$= \frac{\phi 0.87 \times 250}{4 \times 1.4}$

$\Rightarrow L_d = 38.82 \phi$.

In compression, $\tau_{bd} = 1.4 \times 1.25$

$= 1.75 N/mm^2$.

$$L_d = \frac{\phi 0.87 f_y}{4 \tau_{bd}}$$

$$= \frac{\phi \times 0.87 \times 250}{4 \times 1.75}$$

$$\Rightarrow L_d = 31.07 \phi$$

IS code Provision for bond :—

(Page-44, 26.2.3)

curtailment of tension reinforcement

Page-26.2.3.3

Positive moment reinforcement,

$$L_d \leq \frac{M_1}{V} + L_o$$

For the reinforced confined by a compressive reaction, In hook case, $L_d \leq 1.3 \frac{M_1}{V} + L_o$

Date – 07/03/2019

Q:- A S/S beam 25cm x 50cm has two bar of dia 20mm, if shear force at centre of support is 110kN & it is working load. Determine anchorage length. Use M_{20}, Fe415, LSM. The effective cover is 35mm.

Sol:)

Given data,

$A_{st} = 2 \times \frac{\pi}{4} \times 20^2$

$= 628.31 \text{ mm}^2$

$\simeq 630 \text{ mm}^2$.

$b = 25\text{cm} = 250\text{mm}$

$d = 50\text{cm} = 500\text{mm}$.

$d' = 35\text{mm}$.

$f_{ck} = 20 \text{ N/mm}^2$.

$f_y = 415 \text{ N/mm}^2$.

$V = 1.5 \times 110 = 165 \text{ kN}$.

$0.36 f_{ck} x_u b = 0.87 f_y A_{st}$

$\Rightarrow x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$

$= \frac{0.87 \times 415 \times 630}{0.36 \times 20 \times 250}$

$\Rightarrow x_u = 126\text{mm}$.

$(x_u)_{lim} = 0.48d$

$= 0.48 \times 465$

$= 223.2\text{mm}$.

$(x_u) < (x_u)_{lim}$ (under Reinforced)

$M = 0.87 f_y A_{st} (d - 0.42 x_u)$

$= 0.87 \times 415 \times 630 (465 - 0.42 \times 126)$

$= 93.73$ kNm.

$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$

$= \frac{\phi \times 0.87 \times 415}{4 \times (1.92 (1.6 \times 1.2))}$

$= 47\phi = 47 \times 20$

$= 940$ mm

Development length $= 940$ mm

— o —

Torsion :—

There are two types of torsion :—

(i) Primary torsion
(ii) Secondary torsion.

Primary torsion :—

Primary / equilibrium torsion are induced by an eccentric loading with respect to shear centre / equilibrium condition in determining the twisting moment.

Secondary or comparability torsion :—

Torsion is induced by need for member undergoes angle of twist to maintain deformation compatibility & resulting twisting moment depends on torsional stiffness on the member.

Design step for torsion :—

Step-1

calculate Vequivalent (Page-75-41.3.1)

$$V_e = V_u + 1.6 \frac{T_u}{b}.$$

Step-2

calculate longitudinal reinforcement (Page-75-

$$M_{e1} = M_u + M_t$$

Where,

$$M_t = T_u \left(\frac{1 + D/b}{1.7} \right)$$

Step-3

calculate transverse reinforcement

$$A_{sv} = \frac{T_u S_v}{b_t d_1 (0.87 f_y)} + \frac{V_u S_v}{2.5 d_1 (0.87 f_y)}$$

$$A_{sv} < \frac{(\tau_{ve} - \tau_c) b S_v}{0.87 f_y}.$$

Step-4

If τ_{ve} is less than τ_c minimum shear reinforcement will be provided $\frac{A_{sv}}{bS_v}$

$$\frac{A_{sv}}{bS_v} \geq \frac{0.4}{0.87 f_y}.$$

Step-5

Maximum Spacing equal to,

(i) x_1

(ii) $x_1 + y_1$

(iii) 300

$\frac{4}{4}$

Which ever is the less.

Step-6

Calculate the side reinforcement

(page-47)

Q:- A RCC beam of 550 mm x 750 mm overall depth is subjected to ultimate shear force of 130 kN, ultimate bending moment 150 & ultimate twisting moment of 50 kNm. Assume M15 & Fe415 steel. Determine the longitudinal & transverse reinforcem

Soln

Given data,

$b = 550\text{mm}$.

$D = 750\text{mm}$.

$f_{ck} = 15\text{ N/mm}^2$.

$f_y = 415\text{ N/mm}^2$.

$V_u = 130\text{ kN}$.

$M_u = 150\text{ kNm}$..

$T_u = 50\text{ kNm}$

(Page-75)

Equivalent shear, $V_e = V_u + 1.6\frac{T_u}{b}$.

$$\Rightarrow V_e = 130 + 1.6 \times \frac{50}{550 \times 10^{-3}}$$

$$\Rightarrow V_e = 275.45\text{ kN}.$$

$$M_t = T_m\left(\frac{1 + D/b}{1.7}\right)$$

$$= 50\left(\frac{1 + 0.750/0.550}{1.7}\right)$$

$$= 69.51\text{ kN-m}.$$

$M_u = 150\ kN\text{-}m > M_t$

Longitudinal reinforcement,

$M_{eq} = M_u + M_t$

$= 150 + 69.51$

$= 219.51\ kNm$.

As $M_u > M_t$, so $M_{eq} = 0$.

$M_u = 0.138 f_{ck} b d^2$.

$\Rightarrow 219.51 = 0.138 \times 15 \times 550 \times d^2$

$\Rightarrow d = \sqrt{\frac{219.51 \times 10^6}{0.138 \times 15 \times 550}}$

$\Rightarrow d = 439.09\ mm$

$\approx 450\ mm$.

The effective depth is less than overall depth, so the design is safe.

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}\right] bd.$$

$$= \frac{0.5 \times 15}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 219.51 \times 10^6}{15 \times 550 \times (450)^2}}\right] \times 550 \times 450$$

$= 1659.63\ mm^2$.

Let us provide 20 mm dia bar,

$$n \times \frac{\pi}{4} \times 20^2 = 1659.63$$

$$\Rightarrow n = 1659.63 \times \frac{4}{\pi} \times \frac{1}{20^2}$$

$$= 5.28$$

$$\approx 6 .$$

Let us Provide 6 no. of 20 mm dia bar

Now to calculate equivalent shear Stress.

$$\tau_v = \frac{V_u}{bd} \qquad \text{(Page-74)}$$

$$= \frac{275.45 \times 10^3}{550 \times 450}$$

$$= 1.11 \text{ N/mm}^2 < 2.5$$

$\tau_v < \tau_{c max}$ (Design is safe)

$$\frac{100 A_{st}}{bd}$$

$$= \frac{100 \times 6 \times \frac{\pi}{4} \times 20^2}{550 \times 450}$$

$$= 0.76$$

$0.75 \quad 0.54$

$0.76 \longrightarrow 0.60$

$1.00 \quad \tau_2$

$$\tau_c = \frac{0.76 - 0.75}{1.00 - 0.75} \times 0.60 + \frac{0.76 - 1.00}{0.75 - 1.00} \times 0.54$$

$$= 0.564 \text{ N/mm}^2$$

$\tau_v > \tau_c$

$$V_{us} = V_u - \tau_c b d$$

$$= 275.45 - 0.54 \times 550 \times 450$$

$$= 141800 \text{ N}$$

$$= 141.8 \text{ kN}$$

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

Let us provide two leg 8mm dia stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2$$

$$= 100.53 \text{ mm}^2$$

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

$$= \frac{0.87 \times 250 \times 100.53 \times 450}{141800}$$

$$= 69.38$$

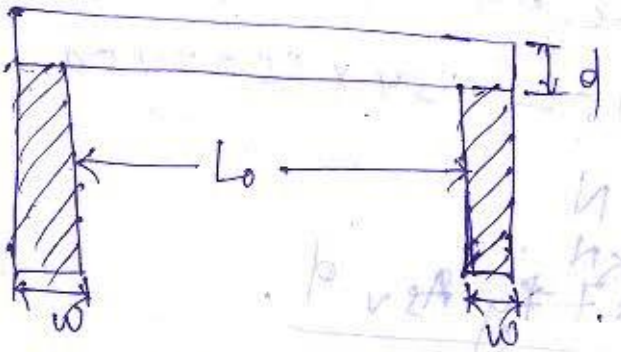
$$\approx 70 \text{ mm}$$

Design of Slab :—

Effective Span :—

(Page-34-22.2)

Simply supported beam :—



Effective Span = clear span + width of the column.

$L_{eff} = L_o + w$ 'or' $L_o + d$.

continuous beam :

case-1

If width of support is (less than) $< \frac{\text{clear span}}{12}$,

$L_{eff} = L_o + w$ 'or' $L_o + d$.

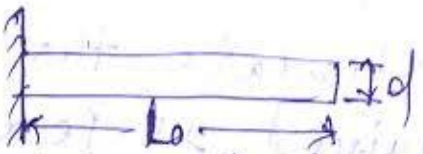
case-2

If width of support $> \frac{\text{clear span}}{12}$,

$L_{eff} = L_{o_3} + \frac{w}{2}$.

L_{o_1} L_{o_2} L_{o_3}

Cantilever / ~~Fixed~~ beam :—



case-1)

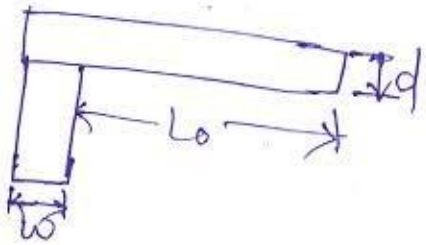
If one end fixed other end free,

$L_{eff} = L_o + \frac{d}{2}$.

case-2

one end continuous, other end free,

$L_{eff} = L_o + \frac{w}{2}$.



Control of deflection :—

(Page-37 ~~93.2~~)

Q:- A Span of S/S beam is 18m. The minimum depth required as per deflection criteria is ?

Ans:—

For simply supported, $\frac{L}{d} \leq 20 \times \frac{10}{L}$.

$\Rightarrow \frac{18}{d} \leq 20 \times \frac{10}{18}$

$\Rightarrow d \geq \frac{18 \times 18}{20 \times 10}$

$\Rightarrow d \geq 1.62$ m.

Q:- calculate the minimum depth required as per deflection criteria for a cantilever span of 7m.

Soln)

For cantilever beam, $\frac{L}{d} \leq 7m$.

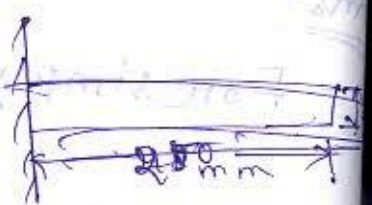
$$\Rightarrow \frac{7}{d} \leq 7$$
$$\Rightarrow d \geq \frac{7}{7}$$
$$\Rightarrow d \geq 1m.$$

Q:- A cantilever beam of span 5m with dimension 250mm x 400mm, check the beam for deflection.

Soln) $L = 5m = 5000mm$.

For cantilever beam,

$$\frac{L}{d} \leq 7$$
$$\Rightarrow \frac{5000}{d} \leq 7$$
$$\Rightarrow \frac{5000}{7} \leq d$$
$$\Rightarrow d = 714.28mm.$$



$d > 400mm$

So the beam is unsafe/failure

Date - 12/03/2019

IS code Provision :-

(i) Minimum reinforcement :- (Page-47)

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

Minimum reinforcement is provided to resist possible load effect & to control cracking in concrete due to shrinkage & temperature variation.

(ii) Maximum tension Reinforcement;

$= 4\%$ of bD

$= 0.04 \times bD$.

(iii) Maximum compression Reinforcement

$= 0.04 \times bD$.

Q:- Why maximum compression Reinforcement is used?

Ans:- To avoid congestion & for Proper Placement & compaction.

Side face Reinforcement :-

→ Side face reinforcement is provided to improve resistance under lateral buckling.

→ cracking can occur on large unreinforced face of concrete on acount of shrinkage & temperature.

Minimum reinforcement of slab

is 0.12% of bD (Fe415 & Fe500)
0.15% of bD (Fe250)

Maximum diameter (Page-48-26.5.2.2)

Maximum diameter of bar should not be greater than (1/8) thickness of the bar

Maximum Spacing :— (26.5.

Maximum spacing should not be greater than equal to (1) 3d or 300mm } for main
(1) 5d or 450mm → Distribution bar

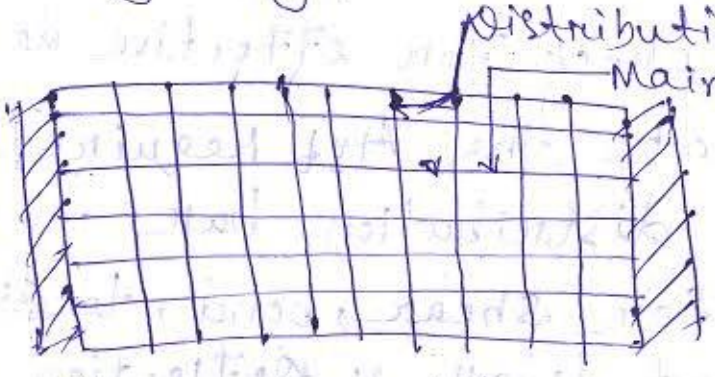
Function of transverse Reinforcement or Distribution bar :—

1. It distributes the effect of point load on the slab more evenly & uniformely.
2. It distributes the shrinkage & temperature crack more uniformely
3. It keeps the main bar in positio

Design of oneway slab :—

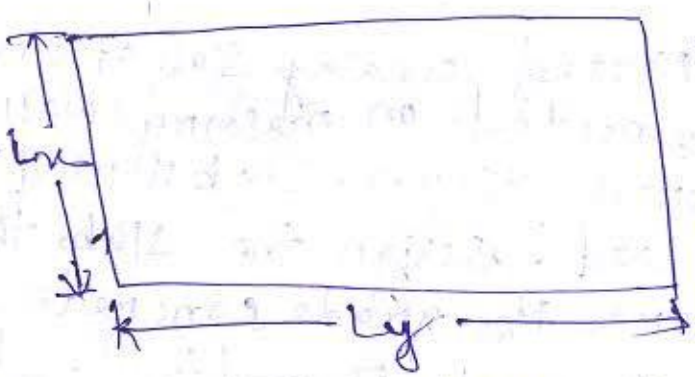
Slab is one way :

1. If supported on opposite side (either supported on shorter edge or longer edge)



(oneway slab)

Page – 90 ~~10.1.11~~



$\frac{L_y}{L_x} < 2 \rightarrow$ Two oneway slab

$\frac{L_y}{L_x} > 2 \rightarrow$ ~~Two~~ one way slab.

Design steps :-

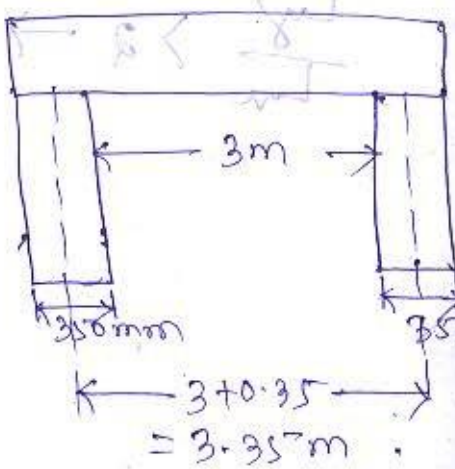
1. Calculate the effective depth from $\frac{L}{d}$ ratio.
2. calculation of ~~dead load~~ DL, L
3. ~~calcul~~ check for effective depth
4. calculate the Ast required.
5. Design distribution bar.
6. check for shear, bond, torsion, development length & deflection.

Date - 14/03/2019

Q:- A simply supported oneway slab of clear span 3m is supported on masonry wall having thickness 350mm. Slab is used for residential load. Design the slab, the materials are M20 grade concrete & HYSD bar of grade Fe415, live load is $2 kN/m^2$ & floor finish is $1 kN/m^2$.

Soln

$f_{ck} = 20 N/mm^2$
$f_y = 415 N/mm^2$.



Page-37

$\frac{\text{Span}}{d} = 20$.

$\frac{3000}{d} = 20$

$\Rightarrow d = \frac{3000}{20} = 150 mm$.

$L_{eff} = 3000 + 150 = 3150\ mm$.

$L_{eff} = 3000 + 350 = 3350\ mm$.

Now, effective Span $= 3150\ mm$.

calculation of shear & moment :—

(Unit weight of concrete $= 25\ KN/m^3$)

Floor finish $= 1\ KN/m^2$

LL of floor $= 2\ KN/m^2$

~~DL of floor $= 0.15 \times 25$~~
~~$= 3.75\ KN/m^2$~~ .

overall depth $= 150 + 15 + \frac{12}{2}$
$= 171\ mm \simeq 180\ mm$.

(Let us provide 12mm dia bar with nominal cover 15 mm)

overall depth $= 180\ mm$.

DL of floor $= 0.180 \times 25$
$= 4.5\ KN/m^2$.

Total load $= 1 + 2 + 4.5$
$= 7.5\ KN/m^2$

Factored load $= 7.5 \times 1.5$
$= 11.25\ KN/m^2$.

For 1m Span the factored load of
$= 11.25 \times 1 = 11.25\ KN/m$.

$S.F. = \frac{wl}{2} = \frac{11.25 \times 3.15}{2}$

$$B.M. = \frac{wl^2}{8}$$

$$= \frac{11.25 \times (3.45)^2}{8}$$

$$= 13.95 \text{ kNm}.$$

$$Mu = 0.138 f_{ck} bd^2$$

$$\Rightarrow 13.95 \times 10^6 = 0.138 \times 20 \times 1000 \times d^2$$

$$\Rightarrow d = \sqrt{\frac{13.95 \times 10^6}{0.138 \times 20 \times 1000}}$$

$$\Rightarrow d = 71.09 \text{ mm}.$$

$(d)_{required} < (d)_{provided}$ (Safe)

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 Mu}{f_{ck} bd^2}}\right] bd$$

$$= \frac{0.5 \times 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 13.95 \times 10^6}{20 \times 1000 \times (150)^2}}\right] \times 1000 \times 150$$

$$\Rightarrow A_{st} = 267.618 \text{ mm}^2$$

Let us provide 12 mm bar

$$A_{st} = \text{No. of bar} \times \frac{\pi}{4} \times 12^2$$

$$\Rightarrow \text{No. of bar} = 267.618 \times \frac{4}{\pi} \times \frac{1}{12^2}$$

$$\Rightarrow n = 2.36$$

Let us provide 4 no. of 12 mm bar.

$$A_{st} = 4 \times \frac{\pi}{4} \times 12^2$$
$$= 452.38 \text{ mm}^2$$

Check for Shear :—

$$D = 180 \text{ mm}$$
$$d = 180 - 15 - 6$$
$$= 159 \text{ mm}$$

$$\tau_v = \frac{V_u}{bd} = \frac{17.72 \times 10^3}{1000 \times 150}$$
$$= 0.118$$
$$(\tau_c)_{max} = 2.8 \text{ N/mm}^2$$
$$\tau_v < (\tau_c)_{max}$$
$$0.118 < 2.8$$

$$p_t = \frac{A_{st}}{bd} \times 100$$
$$= \frac{452.38}{1000 \times 150} \times 100$$
$$= 0.301$$

0.25	—	0.36
0.301	—	?
0.50	—	0.48

$$\tau_c = \frac{0.301 - 0.25}{0.50 - 0.25} \times 0.48 + \frac{0.301 - 0.50}{0.25 - 0.50} \times 0.36$$
$$= 0.384$$

Date - 15/03/

$\frac{A_{sv}}{bS_v} \geq \frac{0.4}{0.87 f_y}$ Page-48

Let us provide 2 leg 6mm dia stirrups

$\frac{2 \times \frac{\pi}{4} \times 6^2}{1000 \times S_v} \geq \frac{0.4}{0.87 \times 250}$

$\Rightarrow S_v \leq \frac{2 \times \frac{\pi}{4} \times 6^2 \times 0.87 \times 250}{0.4 \times 1000}$

$\Rightarrow S_v \leq 30.74 \text{ mm}^2$.

Let us provide two leg 6mm dia stirrup with 25 mm c/c spacing.

Check for deflection :-

Page-37-23.2.1

$\frac{\text{Span}}{d} = \frac{3000}{150}$

$= 20 \text{ mm}$

(OR) $\frac{3000}{159} = 18.86 \text{ mm}$.

As the deflection value is within the limit so the slab is safe in defle

Design of two way slab :-

Q:- A drawing room of a residential building measures 4.3m x 6.55m. It is supported on 350mm thick wall on all 4 sides. The slab is simply supported at edges with no provision to resist torsion at corners. Design the slab using M20 grade concrete & HYSD reinforcement of grade Fe415

Q:- Design a slab of size 4.6m & 5.5m. The slab is continuous over 2 adjacent edges & other two edges are discontinuous. The slab is subjected to live load of $8 KN/m^2$ & floor finish thickness is 100mm is required for water proofing of slab. Design the slab using M25 grade concrete & Fe415. The slab is supported of 300mm wide support.

Date - 16/03/2019

2. Soln

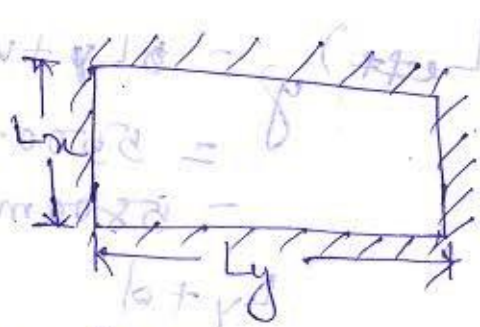
Given data,

$L_x = 4.6 m$

$L_y = 5.5 m$

$f_{ck} = 20 N/mm^2$

$f_y = 415 N/mm^2$.



Page - 39

$\frac{L}{d} = 40 \times \frac{3.5}{4.6}$

$= 30.43 \simeq 32$

$\frac{span}{D} = 32$

$\Rightarrow \frac{4600}{D} = 32$

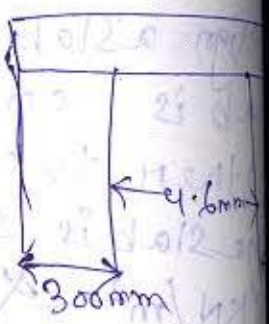
$\Rightarrow D = \frac{4600}{32}$

$\Rightarrow D = 143.75\ mm$.

$\simeq 150\ mm$.

~~Let us~~

$d = 150 - 30 = 120\ mm$.



$Left = L_0 + w$

$= 4600 + 300$

$= 4900\ mm$

$= 4.9\ m$.

or,

$Left = L_0 + d$

$= 4600 + 120$

$= 4720\ mm$

$= 4.720\ m$.

$(Left)_x = 4.72\ m$.

$(Left)_y = L_y + w$

$= 5500 + 300$

$= 5800\ mm = 5.8\ m$

$L_y + d$

$= 5500 + 120$

$= 5620\ mm$

$= 5.62\ m$

$(Left)_y = 5.62\ m$.

check

$\frac{(L_{eff})_y}{(L_{eff})_x} = \frac{5.62}{4.72} = 1.19 < 2$.

(Two way slab).

$W_{DL} = 25 \times 0.15 \times 1 = 3.75$ kN/m.

$W_{LL} = 8 \times 1 = 8$ kN/m.

Unit weight of floor finish = 24 kN/m³

$W_{FL} = 24 \times 0.1 \times 1 = 2.4$ kN/m.

Total Load = 14.15 kN/m.

Factored Load = 14.15×1.5

= 21.225 kN/m.

(Page-91-Table-26)

As per table no. 26, $\alpha_x(-) = 0.060$.

$\alpha_x(+) = 0.045$

$\alpha_y(-) = 0.047$

$\alpha_y(+) = 0.035$

$M_x(-) = 0.060 \times 21.225 \times (4.72)^2$

= 28.37 kNm.

$M_x(+) = 0.045 \times 21.225 \times (4.72)^2$

= 21.28 kNm.

$M_y(-) = 0.047 \times 21.225 \times (4.72)^2$

= 22.22 kNm.

$My(+) = 0.035 \times 21.225 \times (9.72)^2$

$= 23.46$ ~~kNm~~ 16.55 kNm

$Mu = 0.138 f_{ck} bd^2$

$\Rightarrow 28.37 \times 10^6 = 0.138 \times 25 \times 1000 \times d^2$

$\Rightarrow d = \sqrt{\frac{28.37 \times 10^6}{0.138 \times 25 \times 1000}}$

$\Rightarrow d = 95.56$ ~~95.56~~ 90.68 mm < 120 mm ~~Safe~~ (State)

The slab is under reinforced & it is Safe.

Date - 18/03/

We know,

$$A_{st} = 0.5 \frac{f_{ck}}{f_y} \left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}\right] bd$$

	A_{st}	Spacing Required $= \frac{1000}{\left(\frac{A_{st}}{\frac{\pi}{4} \times 10^2}\right)}$
$M_u(+) = 21.28$	530.31	
$M_u(-) = 28.37$	728.57	
$My(+) = 16.55$	404.85	
$My(-) = 22.22$	555.85	

$$\text{Spacing} = \frac{1000}{\text{No. of bar}}$$

$$= \frac{1000}{\left(\frac{A_{st}}{\frac{\pi}{4} \times \phi^2}\right)}$$

Let us Provide 10mm dia bar

	Ast	Spacing required	Spacing Provided
Mx(+) = 21.28 →	530.31 →	$\frac{1000}{\left(\frac{530.31}{\frac{\pi}{4}\times 10^2}\right)} = 148.1$ mm	140mm
Mx(−) = 28.37 →	728.56 →	$1000/\left(\frac{728.56}{\frac{\pi}{4}\times 10^2}\right) = 107.81$mm	100mm
My(+) = 16.55 →	404.85 →	$1000/\left(\frac{404.85}{\frac{\pi}{4}\times 10^2}\right) = 193.99$mm	190mm
My(−) = 22.22 →	555.85 →	$1000/\left(\frac{555.85}{\frac{\pi}{4}\times 10^2}\right) = 141.25$mm	140mm

Area of distribution bar is (Pag-48 26.5.2.1)

$\frac{0.12}{100} \times 1000 \times 150 = 180\ mm^2$

Let us provide 8mm bar as distribution steel.

Spacing between the distribution bar $= \frac{1000}{\frac{Ast}{\frac{\pi}{4}}}$

Spacing between the distribution bar are $\frac{1000}{\left(\frac{180}{\frac{\pi}{4}\times 8^2}\right)} = 279.25$ mm.

$\simeq 270$ mm.

Check for shear :—

$V_x = \frac{WL_x}{3}$ $\quad V_y = \frac{WL_x r}{2+r}$

$= \frac{21.225 \times 4.72}{3}$ $\quad V_y = \frac{21.225 \times 4.72 \times 1.2}{2+1.2}$

$= 33.394$ kN. $\quad = 37.57$ kN.

$$\tau_v = \frac{V}{bd}$$

$$= \frac{37.57 \times 10^3}{1000 \times 190} = 0.313 \text{ MPa}$$

$(\tau_c)_{max} = 3.1 \text{ N/mm}^2$

$\tau_v < (\tau_c)_{max}$ (safe)

check for bond :—

$$\tau_{bd,y} = \frac{V_u}{\Sigma o j d \times \text{no. of bar}}$$

Σo = circumference $= 2\pi r / \pi d$

d = Effective depth

j = Modification factor $= 0.8$

V_u = Shear force

$$\tau_{bd,y} = \frac{33.39 \times 10^3}{\pi \times 10 \times 0.8 \times 120 \times \left(\frac{1000}{190}\right)}$$

$$= 2.1 \text{ MPa}$$

$$\tau_{bd,x} = \frac{V_y}{\Sigma o j d \times \text{no. of bar}}$$

$$= \frac{37.57 \times 10^6}{\pi \times 10 \times 0.8 \times 120 \times \left(\frac{1000}{140}\right)}$$

$$= 1.74 \text{ MPa}$$

$\tau_{bd} = 1.4 \times 1.6 = 2.24$ MPa.

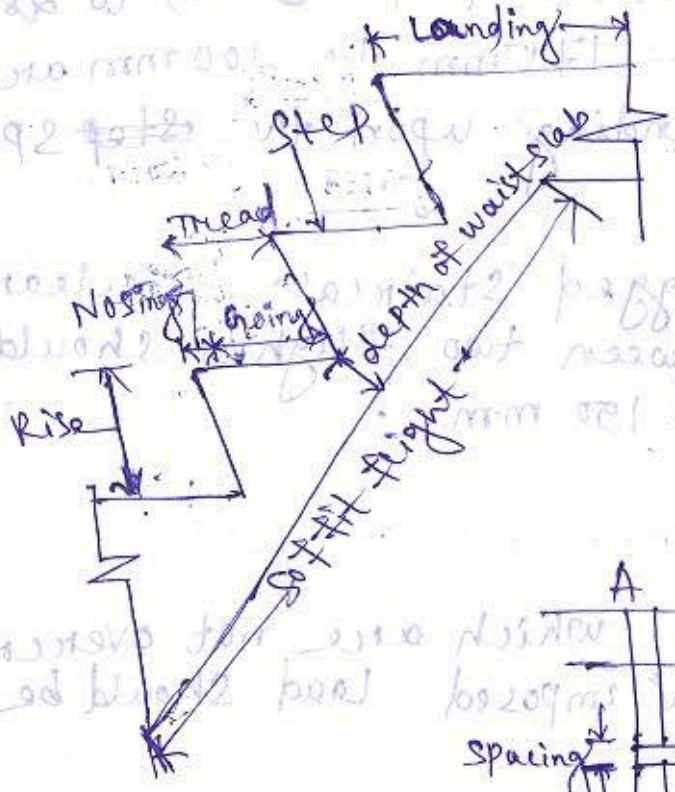
τ_{bd} provide $<$ τ_{bd} permissible.

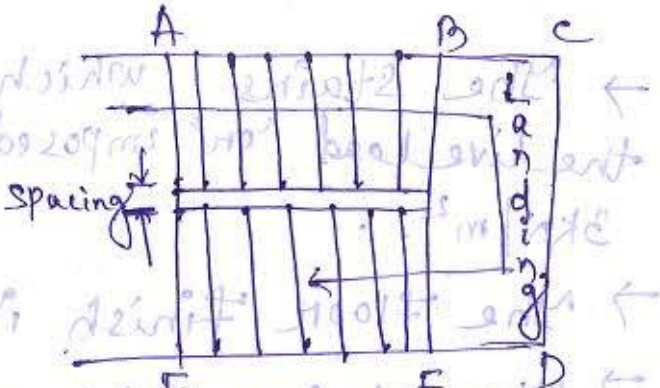
So, it is safe.

Hence all the reinforcement of two support has been considered for bond check. so no cuttlement of bar is required.

Date - 19/03/2019

Design of staircase :-





(Dogregged Stair)

The stairs are grouped into the following categories as per their use:

1. Private stairs.
2. common stairs.

1. Private Stair :—

→ For Private stair the rise should not be more than 200mm & tread is not less than 230 mm.

→ These are minimum requirements & usually a tread of 250mm to 280mm & a rise of 175mm to 200 mm are provided depending upon the ~~step~~ space available.

→ For dog-legged staircase the clear distance between two flights should be between 0 to 150 mm.

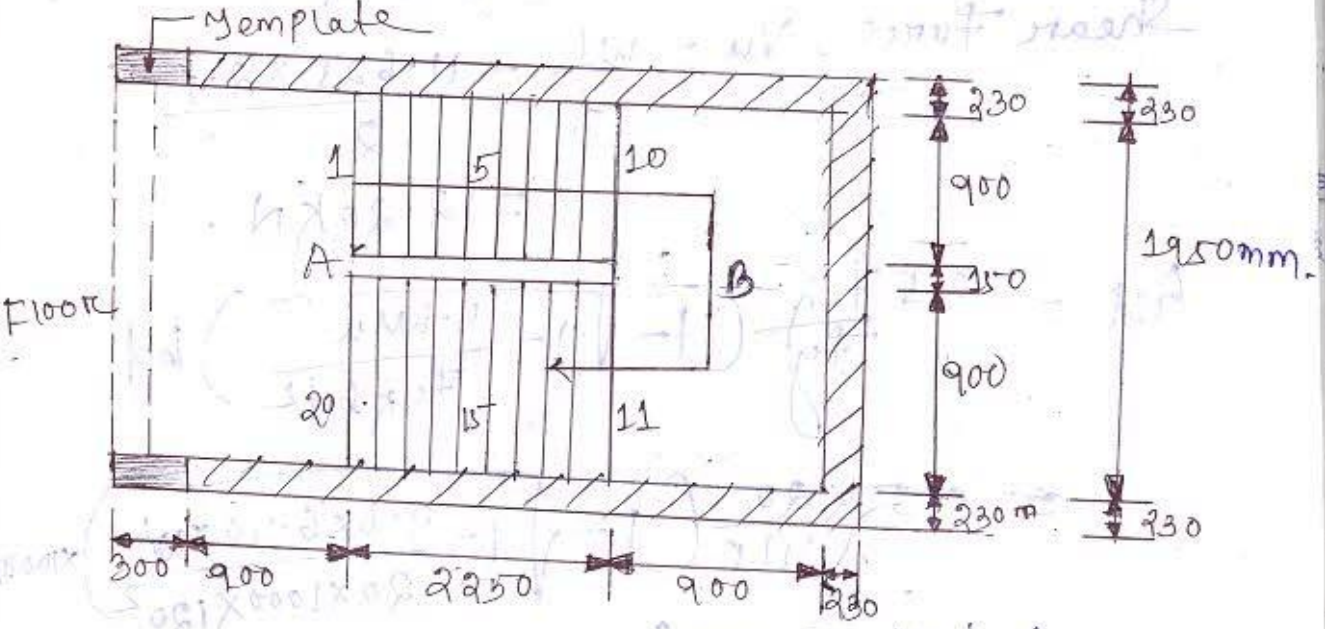
Design requirements for stairs :—

→ The stairs which are not overcrowded the live load or imposed load should be $3KN/m^2$.

→ The floor finish is $1KN/m^2$.

→ The stairs which are overcrowded the live load should be $5KN/m^2$.

Q:- The arrangement of a dog-legged staircase in a residential building is shown in the figure. Rise of step is 160mm & tread is 250mm, nosing is not provided. The materials are M_{20} grade concrete & HYSD bar of Fe415. Design the staircase.



Date - 28/03/2019

Soln

Let us assume the thickness of waist slab is 150mm.

DL = $0.15 \times 25 = 3.75$ kN/m^2.

LL = 3 kN/m^2

FF = 1 kN/m^2.

Total load = 7.75 kN/m^2.

Factored load = $1.5 \times 7.75 = 11.625$ kN/m^2.

The span length = 1950 + 150 = 2100 mm
= 2.1 m.

For 1m span the total load is $11.625 \times 1m$
$= 11.625\ kN/m$.

Moment, $Mu = \frac{wl^2}{8} = \frac{11.625 \times (2.1)^2}{8}$
$= 6.40\ kN-m$.

Shear force, $Vu = \frac{wl}{2} = \frac{11.625 \times 2.1}{2}$
$= 12.20\ kN$.

$$Ast = \frac{0.5 fck}{fy}\left(1 - \sqrt{1 - \frac{4.6 Mu}{fck\, bd^2}}\right) bd$$

$$= 0.5 \times \frac{20}{415}\left(1 - \sqrt{1 - \frac{4.6 \times 6.40 \times 10^6}{20 \times 1000 \times 120^2}}\right) \times 1000 \times 120$$

$= 151.77\ mm^2$
$\approx 152\ mm^2$.

$D = 150mm$.

Let us assume,
clear cover = 25mm. Provide 10mm dia bar.
$\phi = 10mm$.

So, $d = 150 - 25 - \frac{10}{2}$
$= 120mm$.

$n \times \frac{\pi}{4} \times 10^2 = 152$

$\Rightarrow n = 152 \times \frac{4}{\pi} \times \frac{1}{10^2}$

$\Rightarrow n = 1.93$

$\simeq 2$ nos.

Let us provide 2nos. of 10mm dia bar.

Check for Shear

$$\tau_v = \frac{V_u}{bd} = \frac{12.20 \times 10^3}{1000 \times 120}$$

$$= 0.10 \text{ N/mm}^2.$$

$\tau_{c\,max} = 2.8 \text{ N/mm}^2$.

$\tau_v << \tau_{c\,max}$ (Safe)

Check for deflection :—

$$\frac{\text{span}}{d} = 20$$

$$\frac{2100}{120} = 17.5 < 20 \text{ (Safe)}$$

Check for development length

Page-44

$$1.3 \times \frac{M_1}{V} + L_o$$

L_o = effective depth (d) or 12ϕ.

$$1.3 \times \frac{6.40 \times 10^6}{12.20 \times 10^3} + 120$$

$= 801.96$

Date - 29/03/201

Design of Flight

The length of waist slab for one ste

$= \sqrt{250^2 + 160^2}$

$= 296.8 mm$.

Assuming 150mm thickness of waist sl

Self load in plan

$DL = \frac{296.8}{250} \times 0.15 \times 25$ 7.1.5

$= 4.452 kN/m^2$.

The floor finish of a single step

~~$= (0.25 + 0.15) \times 1 = 0.41 kN/m$~~ .

$= \frac{410}{250} \times 1 = 1.64 kN/m^2$

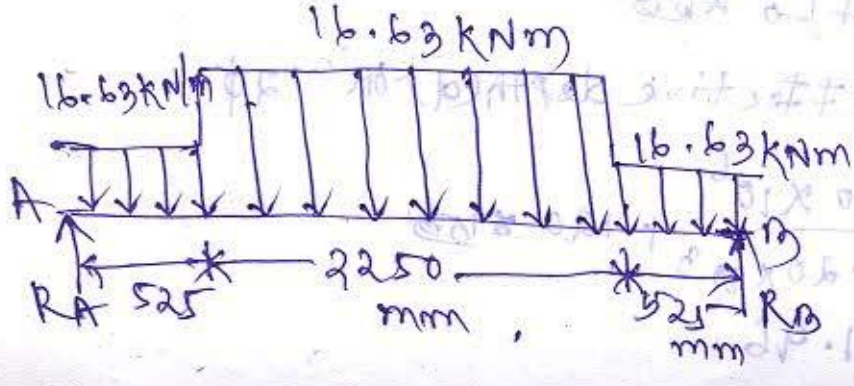
~~base load~~ $LL = 3 kN/m^2$.

Let us assume the self weight of each step is $2 kN/m^2$.

Total load = $11.092 kN/m^2$.

Factored load $= 11.092 \times 1.5$

$= 16.638 kN/m^2$



Shear force = 16.63×2.25

$= 37.41\ kN$.

$11.63 \times 0.525 = 6.105\ kN$.

$6.105\ kN$

$49.62\ kN$.

$R_A + R_B = 49.62$.

$R_A = R_B$

$2R_A = 49.62$

$\Rightarrow R_A = \frac{49.62}{2}$

$R_B = R_A = 24.81\ kN$

Column

Date – 30/03/2019

A column may be classified based on the following criteria:

1. Shape of cross section.
2. Slenderness ratio.
3. Type of loading.
4. Pattern of lateral reinforcement.

Column may be classified based on the types of loading:

1. Axially loaded column.
2. A column subjected to axial loading & uniaxial bending.
3. A column subjected to axial loading & biaxial bending.

The reinforced concrete column can also be classified according to the manner in which the longitudinal bars are laterally supported that is:

1. Tied column
2. Spiral column

Effective height of the column:

(Page-94-Table-28)

Minimum eccentricity :—

(Page-42 - 25.4)

$$e \geq \frac{l}{500} + \frac{D}{30}$$

$$> 20 \text{ .}$$

Short column under axial compression :-

(Page-71-39.3)

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc} \text{ .}$$

Requirements for reinforcement :—

There are two kind of reinforcement in a column :

1. Longitudinal reinforcement.
2. Transverse reinforcement.

Q. The purpose of transverse reinforcement is to hold the vertical bars in position providing lateral support, so that the individual bars cannot buckle outward & split the concrete.

→ The transverse reinforcement does not contribute to the strength of the column directly.

Longitudinal Reinforcement :—

(Page-48- 26.5.3)

→ Minimum percentage of steel is 0.8%.

→ Maximum percentage of steel is 4%, if bars are lapped & 6%, if the bars are not lapped.

→ Minimum no. of bars for a rectangular section is 4 & for circular section it is 6.

→ Minimum diameter of bar is 12mm.

→ Maximum spacing between longitudinal bar is 300mm.

→ ~~Maximum~~ Minimum percentage of steel for pedestal is 0.15%.

→ Minimum nominal cover is 50mm.

Transverse Reinforcement :—

Dia of tie or ring / spiral should that not be ~~less than~~ less than

- $\frac{\phi_{main}}{4}$
- 6 mm

which ever is more

Page-49

Date - 02/04/2019

Q:- Design a short column square in section to carrey an axial load 2000KN using
(i) mild steel
(ii). HYSD bar of Fe415 & the grade of concrete is M_{20}.

Sol:- Axial factored load,

$P_u = 2000KN \times 1.5 = 3000KN = 3000 \times 10^3 N$.

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$.

$A_{sc} = 0.8\%$.

For economy quantity of steel should be adopted, $A_{sc} = 0.8\% = 0.008$

Let us consider the square column has side a.

The area of square column $= a^2$.

~~$P_{uo} = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$~~

~~3000×10^3~~

$A_{sc} = 0.008 a^2$

$A_c = a^2 - 0.008 a^2$

$= a^2 (1 - 0.008)$

$P_u = 0.4 \times f_{ck} \times A_c + 0.67 f_y A_{sc}$

$\Rightarrow 3000 \times 10^3 = 0.4 \times 20 \times a^2 (1 - 0.008) + 0.67 \times 250 \times 0.008 a^2$

$\Rightarrow 3000 \times 10^3 = 7.936 a^2 + 1.34 a^2$

$\Rightarrow 9.276 a^2 = 3000 \times 10^3$

$\Rightarrow a = \sqrt{\frac{3000 \times 10^3}{9.276}}$

$\Rightarrow a = 568.69 mm$

$= 56.86 cm \simeq 60 cm$.

$(a^2)_{provided} = 60 \times 60 = 3600 cm^2$.

$(a^2)_{required} = 56.86 \times 56.86$

$= 3233.05 cm^2$.

$\simeq 3234 cm^2$.

The longitudinal reinforcement 0.8% of area required.

$A_{sc} = 0.008 \times 3234$

$= 25.87 cm^2$.

Let us provide 20 mm bar

$n \times \frac{\pi}{4} \times 20^2 = 25.87$

$\Rightarrow n = 25.87 \times \frac{4}{\pi} \times \frac{1}{20^2}$

$\Rightarrow n = 8.23$

Let us provide 8 nos. of 20mm bar,

So Area of Steel, $A_{sc} = 8 \times \frac{\pi}{4} \times 20^2$

$= 25.13 cm^2$.

Diameter & pitch of later ties

$\phi = \frac{\phi_L}{4} = \frac{20}{4} = 5mm$,

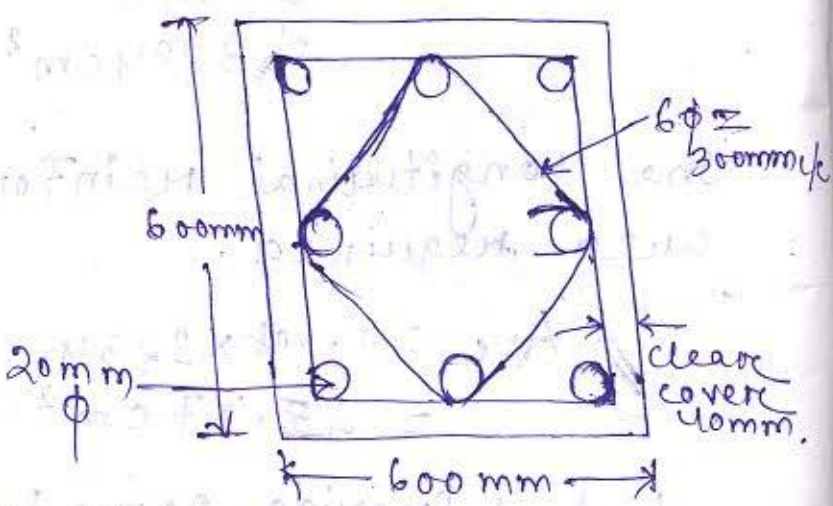
6mm

Let us provide 6mm diameter for lateral ties.

Pitch = 600 mm.

less { $16\phi_L = 16 \times 20 = 320mm$
300 mm

Let us Provide 300 mm pitch c/c.



$P_u = 0.4 f_{ck} \times A_c + 0.67 f_y A_{sc}$

$\Rightarrow 3000 \times 10^3 = 0.4 \times 20 \times a^2 (1 - 0.008) + 0.67 \times 415 \times 0.008 a^2$

$\Rightarrow 3000 \times 10^3 = 7.936 a^2 + 2.2244 a^2$

$\Rightarrow 10.16 a^2 = 3000 \times 10^3$

$\Rightarrow a = \sqrt{\frac{3000 \times 10^3}{10.16}}$

$\Rightarrow a = 543.39 \text{ mm}$

$\simeq 54.33 \text{ cm}$

$\simeq 55 \text{ cm}$.

$(a^2)_{\text{provided}} = 55 \times 55 = 3025 \text{ cm}$.

$(a^2)_{\text{required}} = 54.33 \times 54.33$

$= 2951.75 \text{ cm}$.

The longitudinal reinforcement 0.8% of area required.

$A_{sc} = 0.008 \times 2951.7$

$= 23.61 \text{ cm}^2$.

Let us Provided 20 mm dia bar,

$$n \times \frac{\pi}{4} \times 2^2 = 23.61$$

$$\Rightarrow n = 23.61 \times \frac{4}{\pi} \times \frac{1}{2^2}$$

$$= 7.51$$

$$\simeq 8 \text{ no.}$$

Let us provide 8 nos. of 20 mm bar,

So, Area of steel, $A_{sc} = 8 \times \frac{\pi}{4} \times 2^2$

$$= 25.13 \text{ cm}^2$$

diameter & Pitch of lateral ties

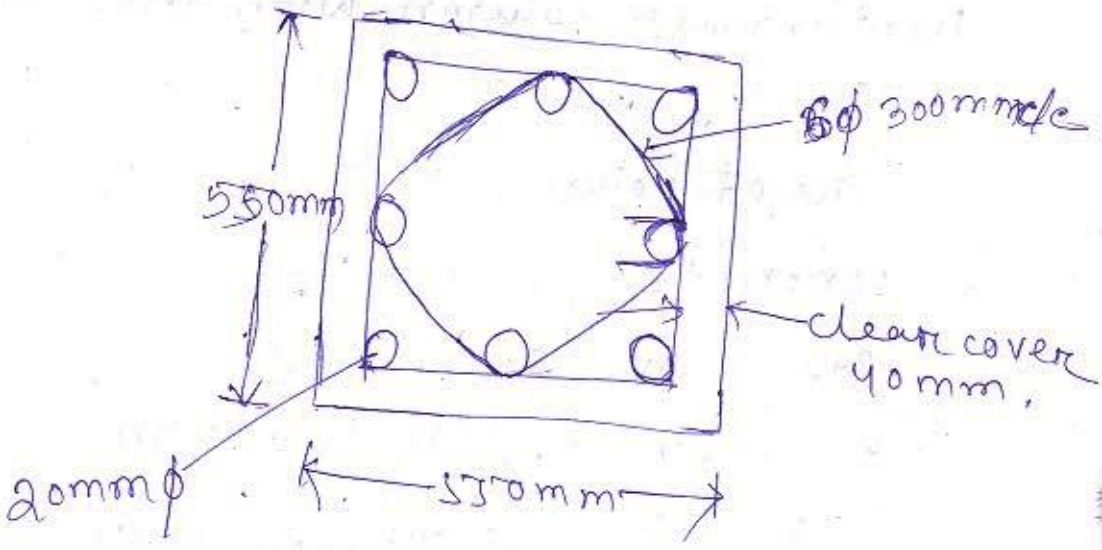
$$\phi = \frac{\phi_L}{4} = \frac{20}{4} = 5 \text{ mm}$$

6 mm

Let us Provide 6 mm diameter for lateral ties:

Pitch = less of
- ~~550~~ mm
- 16 ϕ_L = 16 × 20 = 320 mm
- 300 mm

Let us provide 300 mm pitch c/c.



Date - 03/04/2019

Design of long column (slender column)

(Page - 71, 39.7 (a))

$\frac{L}{D} < 12$ (short column)

$\frac{L}{d} \geq 12$ (long column)

$P_b = (q_1 + \frac{q_2 p}{\sigma_{ck}}) \sigma_{ck} b D$ (Rectangular section)

$P_b = \left(q_1 + \frac{q_2 p}{\sigma_{ck}}\right) \sigma_{ck} D^2$ (Circular sec^n)

q_1 & q_2 are the co-efficient.

Q:- Design a ~~short~~ slender unbraced rectangular column with the following data.

Size of column = 225 cm x 30 cm.

concrete grade = M_{25}

Steel grade = Fe500

Effective length, L_{ex} = 3m.

Effective length, L_{ey} = 4m

Factored load, P_u = 750 kN

Factored moment in the direction of larger dimension M_y = 25 kNm.

Factored moment in the direction of shorter dimension M_x = 15 kNm.

The reinforcement is distributed equally to the all four sides.

$d_1 = 0.2$, $d_2 = 0.34$

Axial load corresponding to the maximum compressive stress is 420 kN.

Sol:-

Given data,

b = 25 cm = 250 mm

D = 30 cm = 300 mm

f_{ck} = 25 MPa

f_y = 500 MPa

L_{ex} = 3m = 3000 mm

L_{ey} = 4m = 4000 mm

P_u = 750 kN = 750×10^3 N.

$M_u = 15\,kNm = 15 \times 10^6\,N\,mm$

$q_1 = 0.2$

$q_2 = 0.34$

$\frac{L_{ey}}{D} = \frac{4000}{300} = 13.33 > 12$ (Long column)

The column is slender about the major axis in the direction of larger dimension only.

~~$M_{ay} = P_u \times e_y$~~

Additional moment $M_{ay} = P_u \times e_y$.

$\Rightarrow 25 \times 10^6 = 750 \times 10^3 \times e_y$

$\Rightarrow e_y = \frac{25 \times 10^6}{750 \times 10^3}$

$\Rightarrow e_y = 33.33\,mm$.

Let us assume percentage of steel is 2%.

Net area $= 250 \times 300$

$= 75000\,mm^2$.

$A_{sc} = 75000 \times 0.02$

$= 1500\,mm^2$

$A_c = 75000 - 1500$

$= 73500\,mm^2$

$P_{uz} = 0.45 f_{ck} A_c + 0.75 f_y \times A_{sc}$

$= 0.45 \times 25 \times 73500 + 0.75 \times 500 \times 1500$

$= 1389.375\,kN \leq 1400\,kN$.

$P_b = 420\,kN = 420 \times 10^3\,N$.

$$K = \frac{P_{uz} - P_u}{P_{uz} - P_b} = \frac{1400 - 750}{1400 - 420}$$

$$= 0.66.$$

The reduction moment $= K \times M_{ay}$

$= 0.66 \times 25$

$= 16.5\,kNm$.

Eccentricity,

$$e \leq \frac{L}{500} + \frac{D}{30} = \frac{4000}{500} + \frac{300}{30}$$

$= 18\,mm$.

Let us provide 20mm eccentricity.

So, eccentricity as per IS code which is 20mm is less than the eccentricity provided in the column section.

So, the design is safe.

$M_u = M_y = P_u \times e = 750 \times 10^3 \times 20$

$= 15 \text{ kNm}$.

M_u & M_y provided is equal to & more than 15 kNm, so it is safe.

Q:- Design a slender ~~braced~~ braced circular column with ~~the following~~ column.

Size of column = 40cm.

Concrete grade = M_{20}.

Steel grade = Fe415.

Effective length = 6m

Unsupported length = 7m.

Factored load, P_u = 1200kN.

Factored moment, M_u = 75kN at top & 50 kNm at bottom.

The column is bent in single curvature.

Sol:-

Given data,

D = 40cm = 400mm.

f_{ck} = 20MPa.

f_y = 415 MPa.

Left = 6m.

Unsupported length = 7000 mm.

$P_u = 1200\,kN$

$P_b = 600\,kN$.

$\frac{L}{D} = \frac{6000}{400} = 15 > 12$ (Long column)

The column is slender about the major axis in the direction of larger dimension only.

Additional moment, $M_{ey} = P_u \times e_y$

$\Rightarrow 75 \times 10^6 = 1200 \times e_y$

$\Rightarrow e_y = \frac{75 \times 10^6}{1200 \times 10^3}$

$= 62.5\,mm$.

Let us assume percentage of steel is 2%.

Net area $= \frac{\pi}{4} \times D^2$

$= \frac{\pi}{4} \times 400^2$

$= 125663.706\,mm^2$

$A_{sc} = 125663.706 \times 0.02$

$= 2513.27\,mm^2$.

$A_c = 125663.706 - 2513.27$

$= 123150.436\,mm^2$

$P_{uz} = 0.45 f_{ck} \times A_c + 0.75 f_y \times A_{sc}$

$= 0.45 \times 20 \times 193150.436 + 0.75 \times 415 \times 2513.27$

$= 1890609.21 \text{ N}$

$= 1890.60 \text{ kN}$.

$\simeq 1900 \text{ kN}$.

$$K = \frac{P_{uz} - P_u}{P_{uz} - P_b}$$

$$= \frac{1900 - 1200}{1900 - 600}$$

$= 0.54$

The reduction moment $= K \times M_{ay}$

$= 0.54 \times 75$

$= 40.5 \text{ kNm}$.

Eccentricity, (page-42)

$$e \leq \frac{L}{500} + \frac{D}{30} = \frac{7000}{500} + \frac{400}{30}$$

$= 27.33 \simeq 30 \text{mm}$

Let us provide 30 mm eccentricity.

So, eccentricity as per IS code which is 30mm is less than the eccentricity provided in the column section.

So, the design is safe.

$M = P_u \times e$

$= 1200 \times 10^3 \times 30$

$= 36$ KNm.

Moment at top ~~&~~ moment at bottom is more than the 36 KNm.

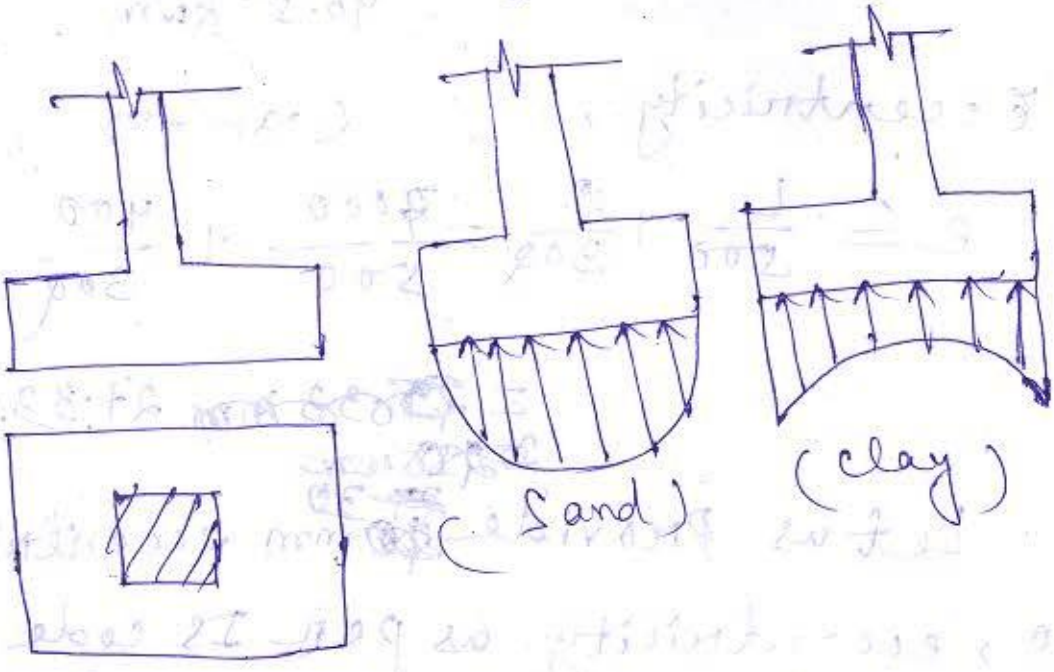
So, it is safe.

Date – 04/04/2019

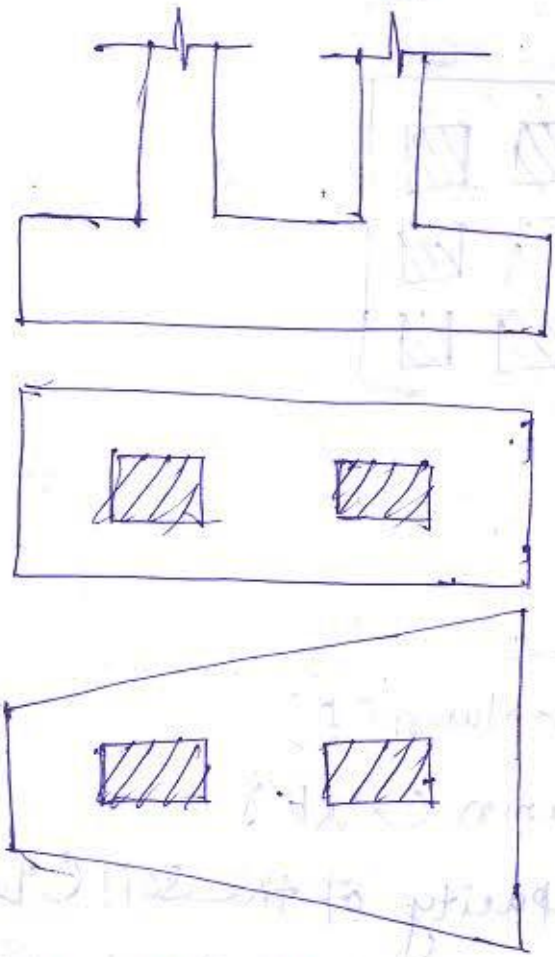
Design of Footing :–

Types of Footing :–

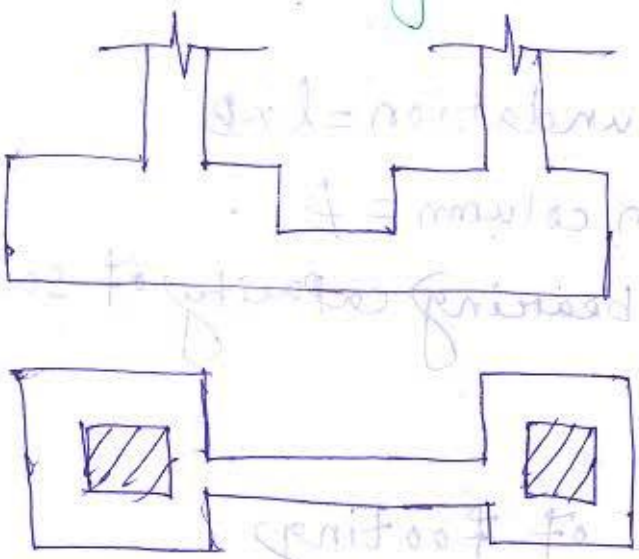
1. Isolated Footing :–

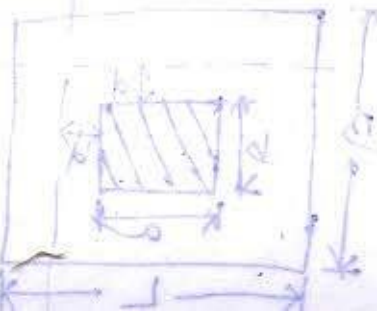


2. combine footing :—

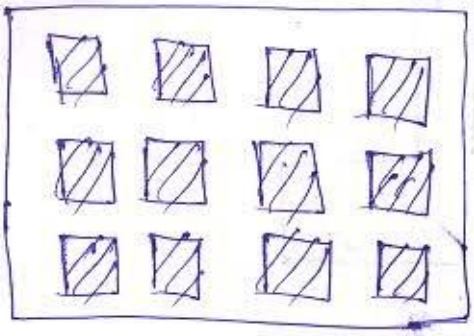


Strap footing :—





Mat footing:-



Design of Rectangular isolated footing:-

Given data

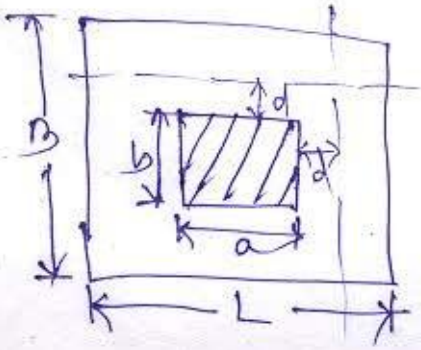
1. Load from column (P)
2. Size of column (a x b)
3. Bearing capacity of the soil (q_o)
4. Grade of concrete & grade of steel.

Step-1

1. Size of foundation = $l \times B$
2. Load from column = P.
3. q_o (gross bearing capacity of soil)

$$= \frac{1.1P}{A}.$$

Where A = Area of footing.



$$\frac{L}{B} = \frac{a}{b}.$$

Net Soil Pressure $w_0 = \frac{1.5P}{A}$.

Step-2

calculation of Bending moment :-

$$M = \frac{wl^2}{8}$$

$$M_{xx} = \frac{w_0(B-b)^2}{8}$$

$$M_{yy} = \frac{w_0(L-a)^2}{8}$$

Step-3

Check for Shear :-

$$V_{xx} = w_0\left[\left(\frac{B-b}{2}\right) - d\right]$$

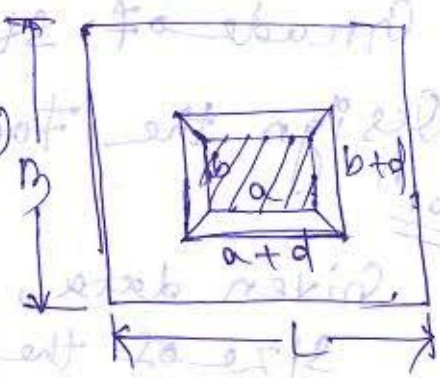
$$V_{yy} = w_0\left[\left(\frac{L-a}{2}\right) - d\right]$$

Step-4

check for punching shear :-

Net Punching force

$F_p = 1.5P + w_0(a+d)(b+d)$



Punching Shear.

$$= \frac{\text{Net Punching force}}{\text{Resisting area}}$$

$$\tau_{vp} = \frac{1.5P + w_0(a+d)(b+d)}{[(a+d)+(b+d)]d}$$

As per IS code,

$$\tau_{vp} = \frac{1.5P - q_o(a+d)(b+d)}{2[(a+d)+(b+d)]d}$$

$\tau_{vp} \not> k\tau_c$

$k = 0.5 + \frac{b}{a} \not> 1$

$\tau_c = 0.25\sqrt{f_{ck}}$

Step-5

Calculation of Area of steel (Ast) :-

$$0.5\frac{f_{ck}}{f_y}\left(1 - \sqrt{1 - \frac{4.6M_u}{f_{ck}bd^2}}\right)bd$$

Date - 05/04/2019

2.

Q:- Design a rectangular isolated ~~beam~~ footing of size 300 x 500mm subjected to a load of 1200KN q_o = 100KN/m.

q_o = Safe bearing capacity.

Grade of concrete is M25.

Grade of steel is Fe415.

Design the footing as per limit state method.

Soln

Given data,

Size of the column = 300x500

P = 1200KN

q_o = 100 KN/m

$f_{ck} = 25\ N/mm^2$

$f_y = 415\ N/mm^2$

$$q_0 = \frac{1.1P}{A_0}$$

$$\Rightarrow A = \frac{1.1P}{q_0}$$

$$= \frac{1.1 \times 1200}{100}$$

$$= 13.2\ mm^2$$

$A = L \times B$

Let us consider, $B = 3m$

$$13.2 = L \times 3$$

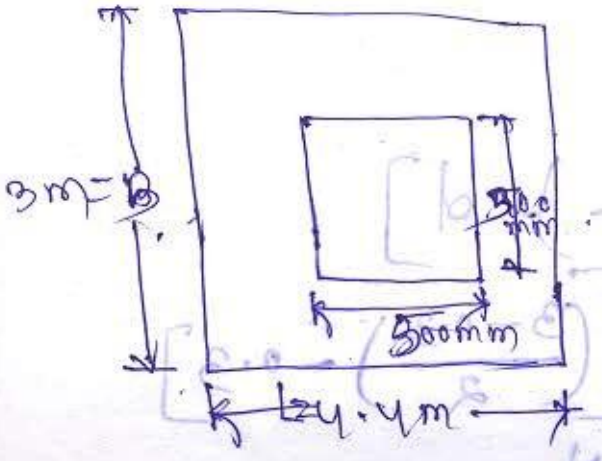
$$\Rightarrow L = \frac{13.2}{3}$$

$$= 4.4m$$

$$w_0 = \frac{1.5P}{A}$$

$$= \frac{1.5 \times 1200}{3 \times 4.4}$$

$$= 136.36\ kN/m^2$$



$$M_{xx} = \frac{W_0 (B-b)^2}{8}$$

$$= \frac{136.36 \times (3-0.5)^2}{8}$$

$$= 284.25 \text{ kNm}$$

$$M_{yy} = \frac{W_0 (L-a)^2}{8}$$

$$= \frac{136.36 \times (4.4-0.5)^2}{8}$$

$$= 259.25 \text{ kNm}$$

$$M_u = 0.138 f_{ck} b d^2$$

$$d = \sqrt{\frac{M_u}{0.138 f_{ck} b}}$$

$$= \sqrt{\frac{259.25 \times 10^6}{0.138 \times 25 \times 1000}}$$

$$= 274.12 \text{ mm}$$

$$\geq 300 \text{ mm}$$

$$V_{xx} = W_0 \left[\left(\frac{B-b}{2}\right) - d\right]$$

$$= 136.36 \times \left[\left(\frac{3-0.3}{2}\right) - 0.3\right]$$

$$= 143.17 \text{ kN}$$

$$V_{yy} = W_o \left[\left(\frac{L-a}{2} \right) - d \right]$$

$$= 136.36 \times \left[\left(\frac{4.4 - 0.5}{2} \right) - 0.3 \right]$$

$$= 224.99 \text{ kN}$$

$$\tau_v = \frac{V_u}{bd}$$

$$= \frac{224.99 \times 10^3}{1000 \times 300}$$

$$= 0.74 \text{ N/mm}^2$$

$(\tau_c)_{max} = 3.1 \text{ N/mm}^2$

$\tau_v < (\tau_c)_{max}$ (Safe)

Check for Punching Shear

$$\tau_{vp} = \frac{1.5P - W_o (a+d)(b+d)}{2[(a+d)+(b+d)]d}$$

$$= \frac{1.5 \times 1200 - 136.36 (0.500+0.3)(0.3+0.3)}{2[(0.5+0.3)+(0.3+0.3)] \times 0.3}$$

$$= 2064.93 \text{ kN/m}^2$$

$$= 2.064 \text{ N/mm}^2$$

$\tau_c = 0.25\sqrt{f_{ck}}$

$= 0.25 \times 5$

$= 1.25 \text{ N/mm}^2$.

$k = 0.5 + \frac{b}{a}$

$= 0.5 + \frac{300}{500}$

$= 1.1$

$k\tau_c = 1.25 \times 1.1$

$= 1.375 \text{ N/mm}^2$.

k value is more than 1

So, it is not safe & $\tau_{vp} > k\tau_c$

So, the design is not safe in Punching Shear.

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left(1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} bd^2}}\right) bd$$

$$= \frac{0.5 \times 25}{415}\left(1 - \sqrt{\frac{4.6 \times 259.25 \times 10^6}{25 \times 1000 \times 300^2}}\right) \times 1000 \times 300$$

$= 2841.42 \text{ mm}^2$/m length.

Let us provide 12 mm bar

$$n \times \frac{\pi}{4} \times (12)^2 = 2841.42$$

$$\Rightarrow n = 2841.42 \times \frac{4}{\pi} \times \frac{1}{12^2}$$

$$\Rightarrow n = 25.12$$

$$\approx 26 \text{ no.}$$

Date - 06/04/2019

Combined footing :-

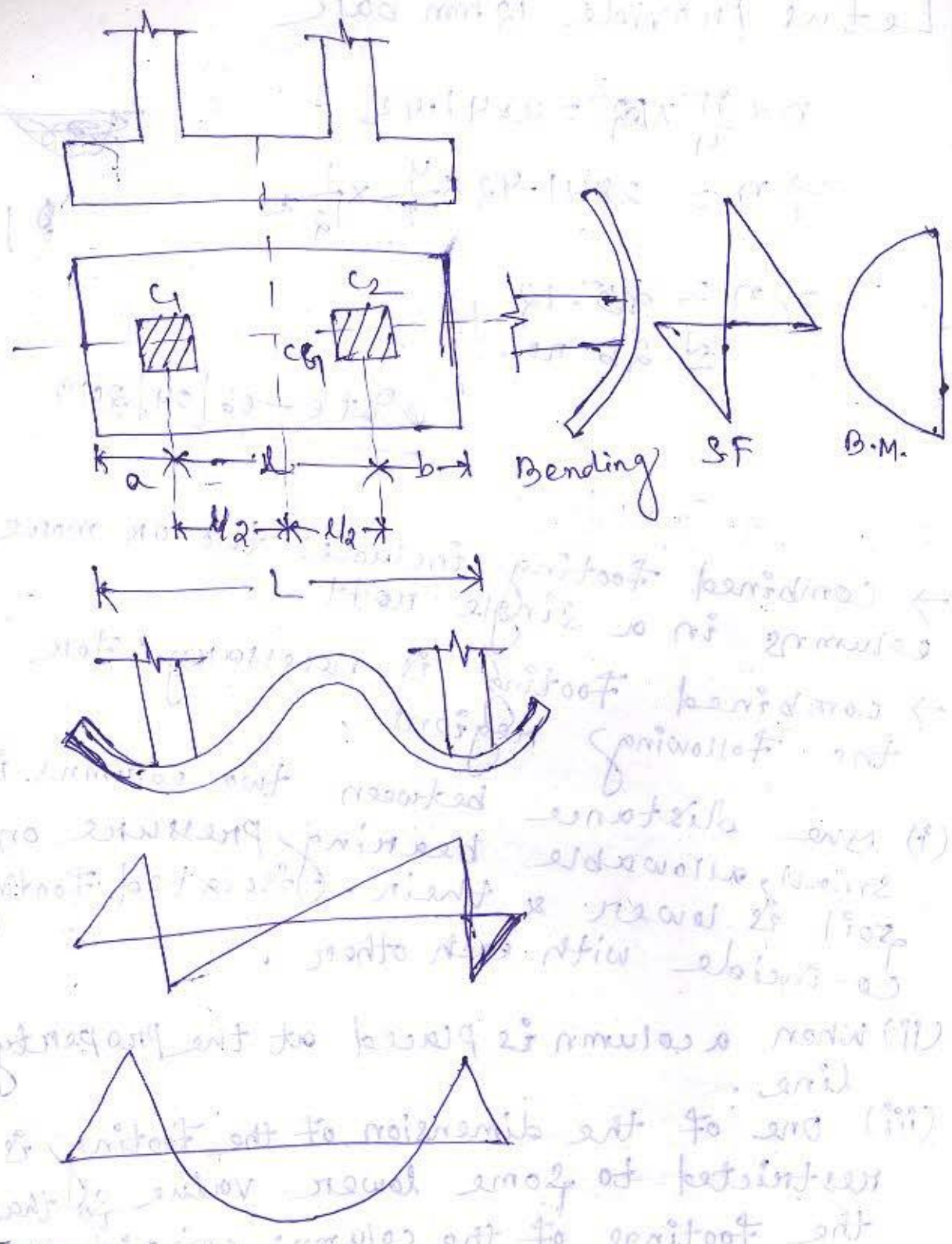
→ Combined footing includes two or more columns in a single raft.

→ Combined footing is necessary for the following regions.

(i) The distance between two columns is small, allowable bearing pressure on soil is lower & their isolated footings co-incide with each other.

(ii) When a column is placed at the property line.

(iii) One of the dimension of the footing is restricted to some lower value so that the footings of the columns coincide with each other.



Q:- Determine the plan & dimension of a combine footing for two axially loaded column with the following data:

(a) width is not restricted.

(b) width is restricted to 2.3 m.

Column	C_1	C_2
	interior	interior
Type		
Size	400 x 400 mm	400 x 400 mm.
P	1000 kN	1000 kN.
Spacing	3 m c/c From C_1 to C_2.	
ABP	150 kN/m^2 at 1.6 m depth.	

Soln

Let us consider self weight of footing be 15% of axial load.

Net Pressure on soil = 2 X 1.15 X 1000

= 2300 kN.

$q_o = \frac{P}{A}$

$\Rightarrow A = \frac{P}{q_o}$

$= \frac{2300}{150}$

$= 15.33 \text{ m}^2$

$P = 1000 \text{ kN}$

$P_f = 1000 \times 15\%$

$= 1000 \times 0.15$

$P + P_f = 1000 + 1000 \times 0.15$

$= 1000 (1 + 0.15)$

$= 1150 \text{ kN}$.

(a) When width is not restricted.

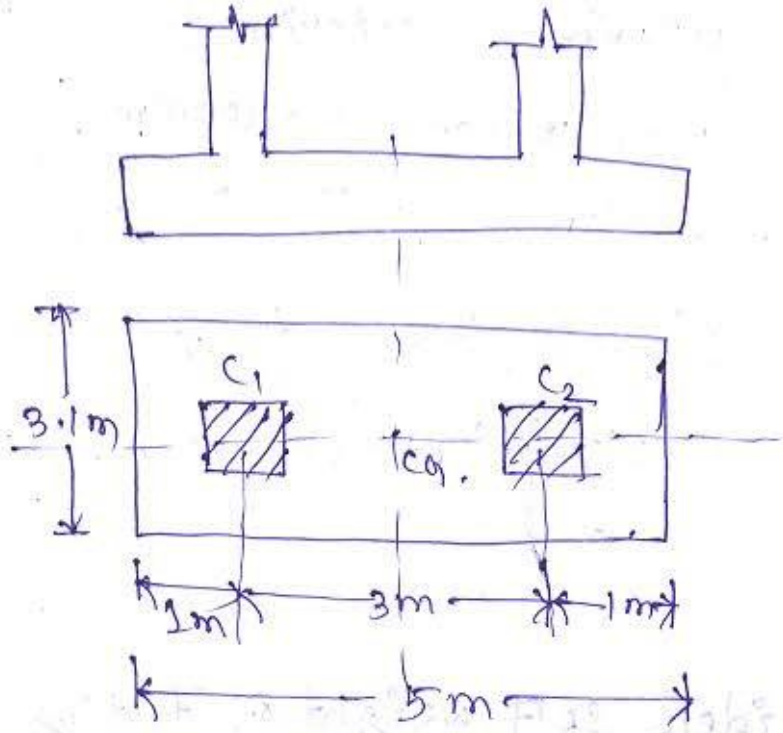
Let us consider 1m projection of both the columns length wise.

$L = 3 + 1 + 1 = 5 \text{ m}$.

$L \times B = 15.33 \text{ m}^2$

$\Rightarrow 5 \times B = 15.33 \text{ m}^2$

$\Rightarrow B = \frac{15.33}{5} = 3.1 \text{ m}$.



(b) When width is restricted to 2.3 m.

$B = 2.3\,m$

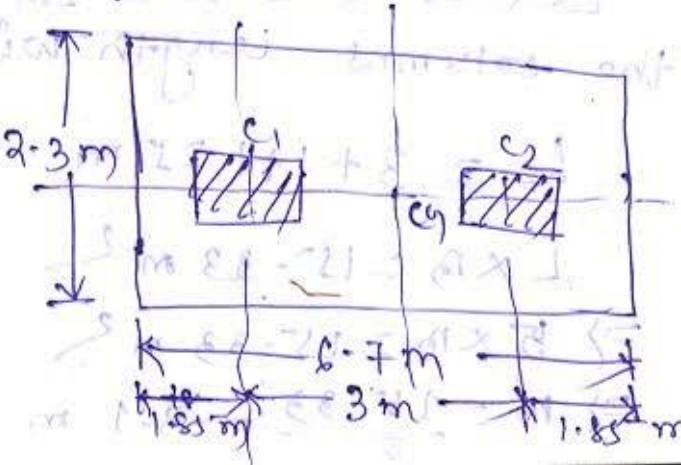
$L \times B = 15.33\,m^2$

$\Rightarrow L = \frac{15.33}{2.3}$

$= 6.7\,m$.

$a + b + l = L \Rightarrow a + b = L - l = 6.7 - 3 = 3.7\,m$.

$\frac{L-3}{2} = \frac{6.7-3}{2} = 1.85\,m$



~~Mood~~ Date - 09/04/2019

Module - IV

Design of Retaining wall

Retaining walls are the structures used to retain earth or other loose material not be able to stand vertically by itself.

Type of retaining wall :—

1. Gravity wall.
2. counterfort wall.
3. cantilever wall.
4. buttress wall.
5. Bridge Abutment.
6. Box culvert.

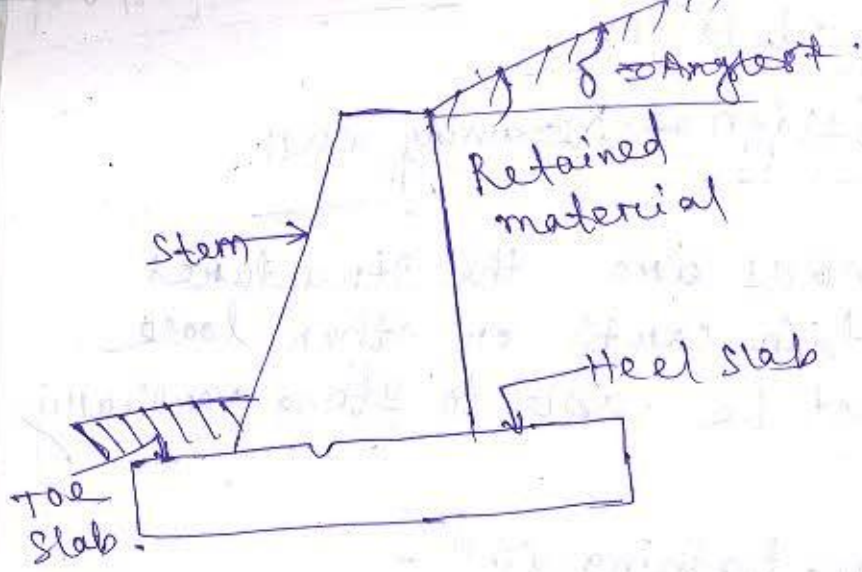
cantilever retaining wall :—

This is the most commonly used retaining wall. It consists of three components ~~first~~

1. vertical walls.
2. ~~Heel~~ Slab.
3. Toe

→ Each of the component act as the cantilever beam

→ Stability is provided by weight of the earth on the base slab & weight of the retaining wall.

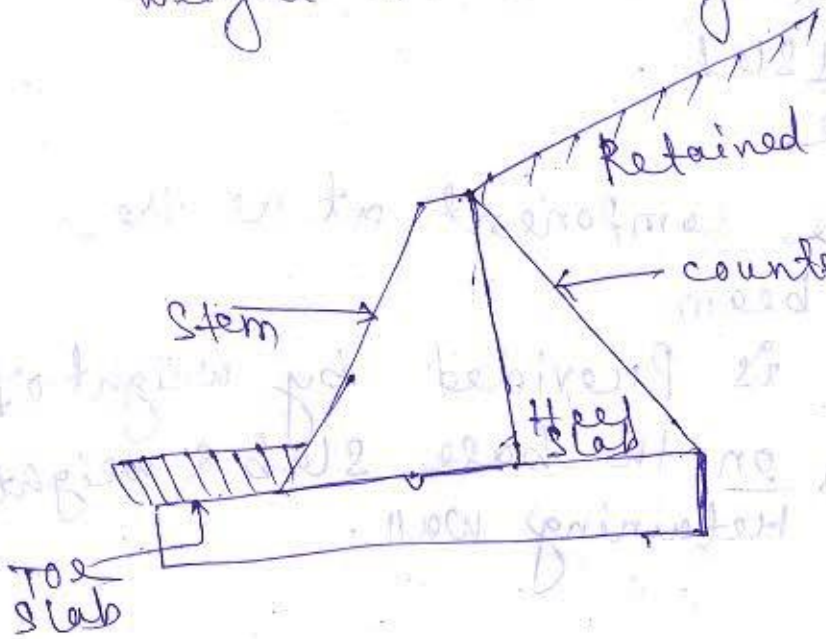


Counterfort retaining wall :—

→ In counterfort retaining wall the vertical slab & horizontal slab, that is hill & toe are tied together by a counterfort.

→ counterforts are transverse walls spaced at certain interval & act as tension ties to support the vertical wall.

→ Stability is provided by weight of the earth on the base slab & the weight of retaining wall.



Forces acting on Retaining wall :—

Generally two types of forces act on retaining wall :

1. ~~Acting~~ Active earth pressure.
2. Passive earth pressure.

$P = k\gamma h$.

Where, P = earth Pressure.

γ = unit weight of retained material

h = Depth of the section below the earth surface.

k = co-efficient of earth pressure. that depends on the properties of soil.

If k_a = co-efficient of active earth pressure.

k_p = co-efficient of passive pressure.

Then, $P_a = k_a \gamma h$.

$P_p = k_p \gamma h$.

P_a = active earth pressure

P_p = Passive earth pressure.

Net active earth pressure on retaining wall = $P_a = \frac{1}{2} k_a \gamma h^2$.

Net passive earth pressure on retaining wall = $P_p = \frac{1}{2} k_p \gamma h^2$.

$$K_a = \frac{1-\sin\phi}{1+\sin\phi}, \quad K_p = \frac{1+\sin\phi}{1-\sin\phi}.$$

ϕ = Angle of repose.

If δ = Angle of surcharged,

then $P_h = P_a \cos\delta$

$P_v = P_a \sin\delta$.

Stability requirement :—

The Stability requirement of the retaining wall has to satisfy the following conditions:

1. Stability against overturning.
2. Stability against sliding.
3. Base width must be adequate to distribute the load to the foundation soil without exceeding the bearing capacity of the soil.

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$$K_a = \cos\delta \left[\frac{\cos\delta - \sqrt{\cos^2\delta - \cos^2\phi}}{\cos\delta + \sqrt{\cos\delta + \cos^2\delta - \cos^2\phi}} \right]$$

$$K_p = \cos\delta \left[\frac{\cos\delta + \sqrt{\cos^2\delta - \cos^2\phi}}{\cos\delta - \sqrt{\cos^2\delta - \cos^2\phi}} \right]$$

Factor of safety for overturning moment

$$= \frac{\text{Resisting moment}}{\text{overturning moment}}$$

$$1 = \frac{0.9 W x_4}{1.4 P_h (H/3)}$$

$$\Rightarrow 1.55 = \frac{W x_4}{P_h (H/3)}$$

where, x_4 = centre of gravity of vertical loads from the toe.

H = depth of the bottom slab below the earth surface.

P_h = Horizontal component of earth pressure.

Factor of safety against sliding :—

$$F.S. = \frac{\text{Resisting force}}{\text{sliding force}}$$

$$1.4 = \frac{0.9 \mu W}{P_h}$$

$$\Rightarrow 1.55 = \frac{\mu W}{P_h}$$

μ = co-efficient of friction between soil & footing.

Proportioning of cantilever wall :—

Height of wall :—

Minimum depth of foundation below ground level should be about 1m. It is necessary to obtain preliminary dimensions of the wall based on certain thumb rule.

Thickness of footing :—

The base thickness is usually 10% of the total height with a minimum of about 30 cm the exact thickness will occurs be governed by the bending moment & shear force consideration.

Thickness of Vertical wall :—

The thickness at top of the wall should not be less than 15 cm. The thickness of vertical wall is determine as required for bending moment & shear force. It may be about 15% of the wall height.

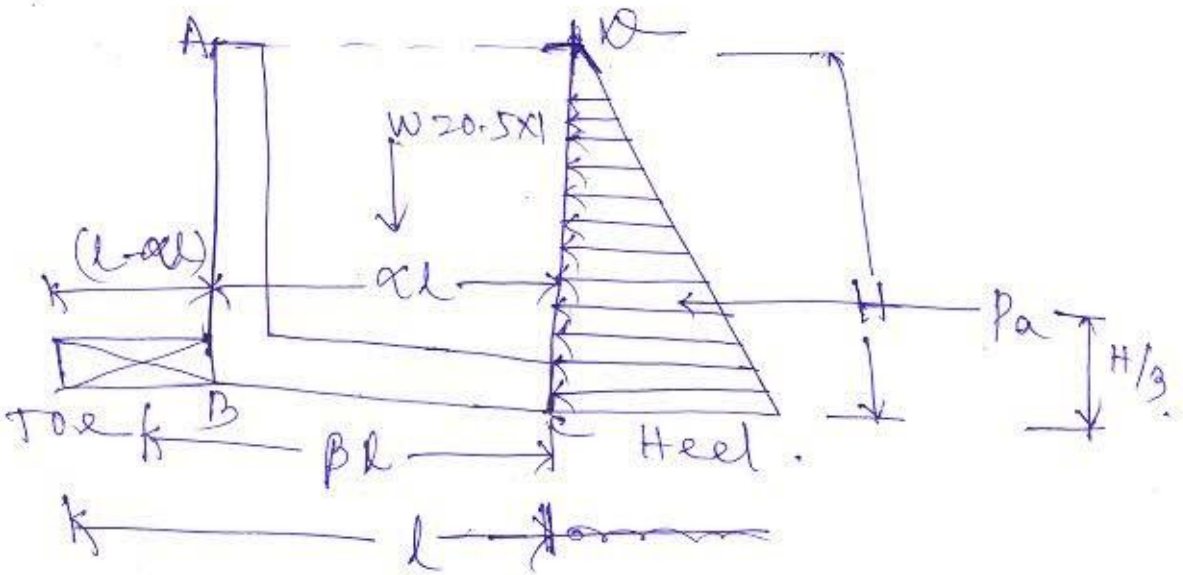
Design of heel :—

Normally the resultant pressure due to downward weight of earth fill & upward bearing pressure is downward which causes tension on the toe face of

Design of toe :—

Normally weight of earth above the toe is neglected & it is design for upward acting bearing pressure as a cantilever beam.

Position of vertical slab on the base footing :—



$$\frac{e}{H} = \sqrt{\frac{K_a \cos\delta}{(1-m)(1+3m)}}$$

m = length of toe

$$q = \frac{\gamma h}{P_s}, \quad 1 - \frac{y}{9q} \text{ if } \delta = 0, \quad 1 - \frac{3}{8q} \; \delta \neq 0.$$

P_s = Bearing capacity of soil length of base.

h = Depth of top of heel slab.

$(M_u)_{lim} = 0.138 f_{ck} b d^2$

$= 0.138 \times 20 \times 230 \times 300^2$

$= 158.7$ kNm

$M_{u2} = M_u - M_{ulim}$

$= 200 - 158.7$

$= 41.3$ kNm

$M_{u2} = f_{sc} A_{sc} (d - d')$

$\Rightarrow 41.3 \times 10^6 = 0.87 f_y A_{sc} (300 - 50)$

$\Rightarrow A_{sc} = \frac{41.3 \times 10^6}{0.87 \times 415 \times 450}$

$= 260 \text{ mm}^2$.

Let us provide 16 mm dia bar,

$n \times \frac{\pi}{4} \times 16^2 = 260$

$\Rightarrow n = 260 \times \frac{4}{\pi} \times \frac{1}{16^2}$

$= 4$ no.

Hence provide 4 nos. 16 mm dia bar

$A_{sc} = 4 \times \frac{\pi}{4} \times 16^2$

$= 804 \text{ mm}^2$

$A_{s2} = \frac{A_{sc} f_{sc}}{0.87 f_y} = 260 \text{ mm}^2$.

$(M_u)_{lim} = 0.87 f_y (A_{st}) (d - 0.42 x_{u,max})$

$\Rightarrow 158.7 \times 10^6 = 0.87 \times f_y$

$\Rightarrow 158.7 \times 10^6 = 0.87 \times 415 \times (A_{st1}) \times (500 - 0.42 \times 240)$

$\Rightarrow A_{st1} = 1101.8\ mm^2$

$= 1110\ mm^2$

$M_{u\,lim} = 0.48 \times 500$

$= 240\ mm$.

$A_{st} = A_{st1} + A_{st2}$

$= 1110 + 260$

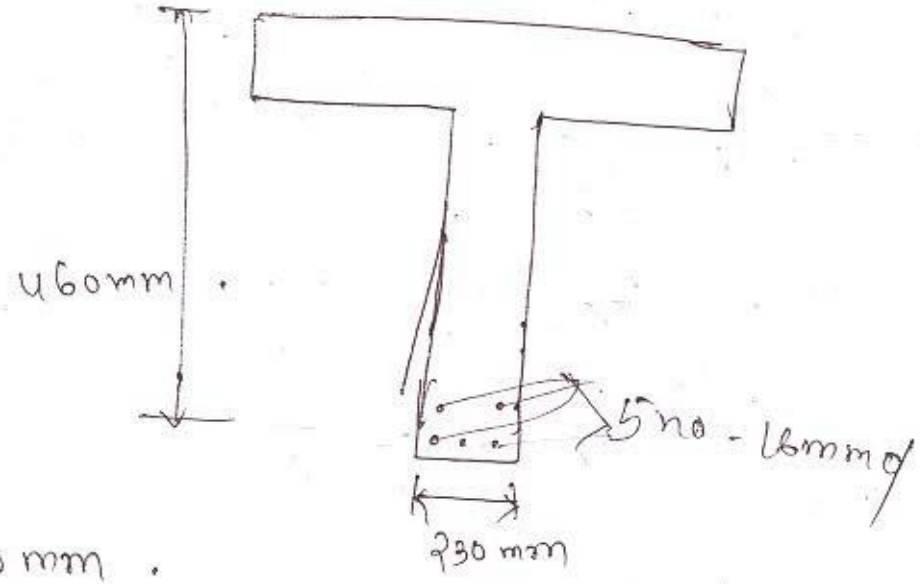
$= 1370\ mm^2$.

Let us provide 20mm dia bar

$\frac{\pi}{4} \times 20^2 = 1370$

$\Rightarrow n = 4.36 \simeq 5\ no.$

$A_{st2} = 5 \times \frac{\pi}{4} \times 20^2 = 1570\ mm^2$.



$b_w = 230\ mm$.

$d = 460\ mm$.

$V_u = 52.5\ kN$.

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$.

$$\tau_v = \frac{V_u}{b_w d} = \frac{52.5 \times 10^3}{230 \times 460} = 0.49\ N/mm^2$$

$\tau_{c\,max} = 2.8\ N/mm^2$.

$\tau_v < \tau_{c\,max}$ Safe

0.75 → 0.56
1.00 0.62

$$P_t = \frac{100 A_s}{bd}$$

$$A_{st} = 5 \times \frac{\pi}{4} \times 16^2 = 1005.31 \approx 1006\ mm^2$$

$$P_t = \frac{100 \times 1006}{230 \times 460} = 0.95$$

$$\tau_c = \frac{0.95 - 0.75}{1 - 0.75} \times 0.62 + \frac{0.95 - 1}{0.75 - 1} \times 0.56$$

$$= 0.608$$

$\frac{\tau_c}{2} = \frac{0.608}{2} = 0.304$

$\tau_v \leq \frac{\tau_c}{2}$.

$\tau_v < \frac{\tau_c}{2}$.

$\frac{\tau_c}{2} < \tau_v < \tau_c$.

Minimum shear reinforcement will be provided.

$$\frac{A_{sv}}{b_{sv}} \geq \frac{0.4}{0.87 f_y}$$

for stirrups, $f_y = 250 N/mm^2$.

Let us provide 2 legged 6mm ϕ stirrups.

$$A_{sv} = 2 \times \frac{\pi}{4} \times 6^2$$
$$= 56.54 mm^2$$
$$\approx 57 mm^2.$$

$$\frac{57}{230 \times s_v} \geq \frac{0.4}{0.87 \times 250}$$

$$\Rightarrow s_v \leq \frac{57 \times 0.87 \times 250}{230 \times 0.4}$$

$$\Rightarrow s_v \leq 134.75 mm.$$

The spacing shall not be exceed

(a) $0.75 \times d = 0.75 \times 460 = 345$ mm.

(b) $S_v = 130$ mm.

(c) 300 mm.

Let us provide 2 leg 6mm dia stirrups with 130 mm c/c spacing.

$V_u = 90$ kN.

$$\tau_v = \frac{V_u}{bd} = \frac{90 \times 10^3}{230 \times 460} = 0.85 \text{ N/mm}^2.$$

$$A_{st} = 5 \times \frac{\pi}{4} \times 16^2 = 1006 \text{ mm}^2.$$

$(\tau_c)_{max} = 2.8$ N/mm^2.

$\tau_c = 0.608$ N/mm^2.

$\tau_v > \tau_c$.

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

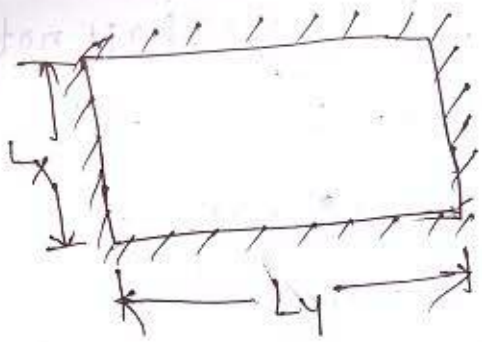
$V_{us} = V_u - \tau_c = 90 \times 10^3 - 0.608$

$= \tau_c b d = 0.608 \times 230 \times 460 = 64326.4$ N

$= 64.32$ kN.

$V_{us} = V_u - \tau_c bd = 90 - 64.32$

$= 25.7$ kN.



$L_x = 4.0\,m$.

$L_y = 5.5\,m$.

$f_{ck} = 25\,N/mm^2$.

$f_y = 415\,N/mm^2$.

$$\frac{L}{D} = 35 \times \frac{3.5}{4}$$

$$= 30.62$$

$$\approx 32$$

$$\frac{L}{D} = 32$$

$$\Rightarrow \frac{4000}{D} = 32$$

$$\Rightarrow D = \frac{4000}{32} = 125\,mm.$$

$$= 130\,mm.$$

$$d = 130 - 30 = 100\,mm.$$

$$(L_{eff})_x = 4.0 + 0.1 = 4.1\,m$$

$$(L_{eff})_y = 5.5 + 0.1 = 5.6\,m.$$

$$\frac{L_y}{L_x} = \frac{5.6}{4.1} = 1.36 \approx 1.4$$

Superimposed load $= 5\,kN/m^2$.

DL of floor $= 25 \times 0.13 = 3.25\,kN/m^2$.

Floor finishing $= 24 \times 0.13 = 3.12\,kN/m^2$.

Total load = 11.37

Factored load = $11.37 \times 1.5 = 17.055\ kN/m^2$

For 1m. $= 17.055\ kN/m$.

As per table-27,

$\alpha_x = 0.099$

$\alpha_y = 0.051$.

$M_{ux} = 0.099 \times 17.055 \times 4.1^2$

$= 28.38\ kNm$.

$M_{uy} = 0.051 \times 17.055 \times 4.1^2$

$= 14.62\ kNm$.

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}\right] bd$$

$M_u = 0.138 f_{ck} b d^2$

$$\Rightarrow d = \sqrt{\frac{28.38 \times 10^6}{0.138 \times 25 \times 1000}}$$

$= 90.69 < 100$ (safe)

Let 1m.

	A_{st}	Spacing	Provide
$M_{ux} = 28.38$	$930\ mm^2$	84	80 mm.
$M_y = 14.62$	$930\ mm^2$	179	170 mm.

Area of distribution bar,

$$\text{Ast} = \frac{0.12}{100} \times 1000 \times 130 = 156 \text{ mm}^2$$

Spa. 8mm ϕ.

$$\text{spacing} = \frac{1000}{\frac{156}{\frac{\pi}{4} \times 8^2}} = 322.21 \approx 300 \text{ mm}$$

check for shear

$$V_u = \frac{w l_n}{3} \qquad V_4 = \frac{w l_n x}{2+x}$$

$$= \frac{17.055 \times 4.1}{3} \qquad = \frac{17.055 \times 4.1 \times 1.4}{2+1.4}$$

$$= 23.31 \text{ KN} \qquad = 28.79$$

$$\tau_v = \frac{28.79 \times 10^3}{1000 \times 100} = 0.28$$

$\tau_{c max} = 3.1$

$\tau_v < \tau_{c max}$ (safe),

check for bond.

$$(\tau_{bd})_u = \frac{V_u}{\Sigma_0 j d \times \text{no. of bar}}$$